**Alternate Examples for SRIS Chapter 1**

***Page 7: Colors of Gummy Bears***

Courtney and Lexi wondered if the distribution of color was the same for name-brand gummy bears (Haribo Gold) and store-brand gummy bears (Great Value). To investigate, they randomly selected six bags of each type and counted the number of each color. Here are the data:

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Name**  **Brand** | **Store**  **Brand** | **Total** |
| **Red** | 137 | 212 | **349** |
| **Green** | 53 | 104 | **157** |
| **Yellow** | 50 | 85 | **135** |
| **Orange** | 81 | 127 | **208** |
| **White** | 52 | 94 | **146** |
| **Total** | **373** | **622** | **995** |

**Problem:** For each brand, do the results in the table summarize a categorical or numerical variable? Explain.

**Solution:** Color is a categorical variable because the outcomes fall into 5 possible categories: red, green, yellow, orange, and white.

**Problem:** Explain why it would be easier to compare the percent of bears of each color rather than the number of bears of each color.

**Solution:** Because the sample of store-brand gummy bears was nearly twice as large as the sample of name-brand gummy bears, we would expect the number of each color to be quite different between the two brands, even if the distribution of color was the same. Using the percent (or relative frequency) of color allows us to compare the brands on the same scale.

**Problem:** For each brand, calculate the percent of gummy bears of each color.

**Solution:** Here are the percents:

|  |  |  |
| --- | --- | --- |
|  | **Name**  **Brand** | **Store**  **Brand** |
| **Red** | 137/373 = 36.7% | 212/622 = 34.1% |
| **Green** | 53/373 = 14.2% | 104/622 = 16.7% |
| **Yellow** | 50/373 = 13.4% | 85/622 = 13.7% |
| **Orange** | 81/373 = 21.7% | 127/622 = 20.4% |
| **White** | 52/373 = 13.9% | 94/622 = 15.1% |

**Problem:** Use the percents from the previous problem to make a bar chart so the distributions of color can be easily compared. Describe what you see.

**Solution:** The distributions of color look approximately the same for the name-brand and store-brand gummy bears. The name brand had a slightly higher percentage of red and orange, while the store brand had a slightly higher percentage of green, yellow, and white.

**Problem:** The following pie chart shows information about the number of red gummy bears for both brands. Explain why this graph doesn’t allow us to make a fair comparison of the two brands.

**Solution:** From the pie chart, it appears that the store brand has a much higher percentage of red gummy bears. However, because the overall sample sizes were so different, comparing the number of red gummy bears for each brand isn’t a fair comparison. In fact, according to the bar graph, the name brand had a slightly higher percentage of red gummy bears.

***Page 16: Does The Weather Channel Deliberately Make Mistakes?***

Nate Silver is one of ESPN’s newest analysts, but he is also famous for making political predictions. In his most recent book (*The Signal and the Noise: Why So Many Predictions Fail – But Some Don’t,* The Penguin Press 2012), Silver claimed that the Weather Channel deliberately makes poor predictions in some instances. People tend to notice the failure to predict rain more than they do other mistakes. People don’t tend to mind an unexpected sunny day, but they very much mind having their picnic ruined by an unexpected rain storm.

When The Weather Channel states that there is a 20% chance of rain, it should actually rain 20% of the time in order for their predictions to be considered “calibrated.” However, a study by Texas A & M University found that out of 113 days that The Weather Channel predicted a 20% chance of rain, it only rained 6 times.

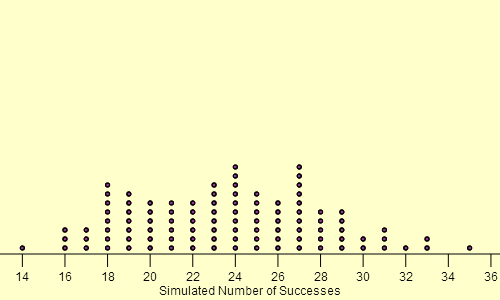
**Problem:** Make a graph to compare the expected percentage of rainy days and the actual percentage of rainy days. Briefly describe what you see.

**Solution:** On days with a 20% chance of rain, we expect it to rain 20% of the time. However, the actual percentage was only 6/113 = 0.053 = 5.3%. This is much lower than expected.

**Problem:** Assuming that it actually rains on 20% of all days when there is a 20% chance of rain, describe how to use a spinner to simulate the number of rainy days in this context.

**Solution:** Make a pie chart with a 20% region representing a rainy day and a 80% region representing a dry day. Because there were 113 days with a 20% chance of rain, spin the spinner 113 times and record the number of rainy days.

**Problem:** One hundred trials of the simulation described above were carried out and the number rainy days was recorded for each trial on the dotplot below. Describe what information is provided by the dotplot.



**Solution:** The dotplot shows the possible number of rainy days that could occur by random chance alone, assuming that it will rain 20% of all days when there is a 20% chance of rain. It could rain as few as 14 days or as many as 35 days, just by chance.

**Problem:** Based on the results of the simulation, is there convincing evidence that The Weather Channel deliberately makes mistakes? Explain.

**Solution:** Yes. Because the observed number of rainy days (6) was well below the smallest value from the simulation, we can essentially rule out random chance as an explanation for The Weather Channel’s poor performance in predicting the rain. We have very convincing evidence that The Weather Channel deliberately makes mistakes.

**Problem:** If had rained in 18 of the 113 days with a 20% chance of rain, would your conclusion be different?

**Solution:** Yes. Even though 18/113 (15.9%) is less than expected, it is possible to get a percentage this low by random chance alone. In the simulation, 15 of the 100 trials resulted in 18 or fewer days with rain. In this case, there is not convincing evidence that The Weather Channel is deliberately making mistakes—the difference between the observed and expected number of rainy days could be due to random chance.

***Page 23: Do Students Prefer Name-brand Cookies?***

A random sample of 30 high school students were asked to taste two unlabeled cookies and identify which cookie they preferred. One of these cookies was a name-brand cookie (Chips Ahoy!) and the other was a store-brand cookie (ChipMates). The order that the two cookies were presented was determined at random using a coin flip. Of these 30 students, 19 preferred the name-brand cookie and 11 preferred the store-brand cookie.

**Problem:** If the two brands of cookies are equally preferred among high school students, what percent would you expect to choose the name-brand cookie?

**Solution:** Because students only tasted two cookies, if they were equally preferred we would expect 1/2 = 50% to choose the name-brand cookie and 50% to choose the store-brand cookie.

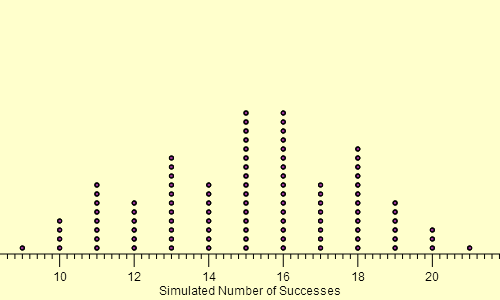
**Problem:** Make a graph to compare the expected percent of students who would choose the name-brand cookie and the actual percent of students who chose the name-brand cookie. Describe what you see.

**Solution:** The actual percent who preferred the name-brand cookie is 19/30 = 63.3%. This is more than the 50% which would be expected if the two brands were equally preferred.

**Problem:** Assuming that 50% of all students would choose the name-brand cookie, describe how to use a random number generator to simulate the selections of 30 students.

**Solution:** Using the numbers 1–100, I will let 1–50 represent name-brand and 51–100 represent store-brand. To simulate the selections of 30 students, generate 30 integers from 1–100 and record how many are between 1 and 50 inclusive.

**Problem:** Here are the results of 100 trials of the simulation described above. Briefly explain what information this dotplot provides.



**Solution:** This dotplot shows the possible number of students who would choose the name-brand cookies by random chance, assuming that 50% of all students would choose the name-brand cookie. As few as 9/30 and as many as 21/30 would choose the name-brand cookie, just by chance.

**Problem:** Based on the results of the simulation, is there convincing evidence that students prefer name-brand cookies? Explain.

**Solution:** No. Even though 19/30 is more than expected, it is possible to get a total of 19 or more simply by random chance. In the simulation, 10 of the 100 trials produced a total of 19 or more. There isn’t convincing evidence that students prefer name-brand cookies.

***Bonus #1: The last chocolate***

Do people have a preference for the last thing they taste? Researchers at the University of Michigan designed a study to find out. The researchers gave 22 students five different Hershey’s Kisses (milk chocolate, dark chocolate, crème, caramel, and almond) in random order and asked the student to rate each one. Participants were not told how many Kisses they would taste. However, when the 5th and final Kiss was presented, participants were told that it would be their last one. Of the 22 students, 14 of them gave the final Kiss the highest rating.

<Source: <http://www.sitemaker.umich.edu/eob/files/obrienellsworth2012.pdf>>

**Problem:** If the participants in the study don’t have a special preference for the last Kiss they try, what percent of the students should pick the last candy?

**Solution:** Because the Kisses were presented in a random order, each student’s favorite type is equally likely to be presented first, second, third, fourth, or last. Thus, about 1/5 = 20% of the participants would be expected to pick the last Kiss.

**Problem:** Based on the results of the study, is there evidence that people do prefer the last one they try? Explain.

**Solution:** Yes. In the study, 14/22 = 64% chose the last kiss. This is much greater than 20%.

**Problem:** Describe how you could use a spinner to simulate the results of this study, assuming that people do not have a special preference for the last Kiss they try.

**Solution:**  Make a spinner with a 20% region for “prefer last” and a 80% region for “not last.” Spin the spinner 22 times and count how often it lands in the “prefer last” segment.

**Problem:** Here are the results of 100 trials of the simulation, assuming that 20% will prefer the last kiss they try. Describe what information is provided by the dotplot.



Simulated number who prefer last Kiss

**Solution:** The dotplot shows the possible number of people that would choose the last kiss, assuming that 20% of all people would prefer the last one they try. There could be as few as 0/22 or as many as 9/22 that will prefer the last one they try, simply by random chance.

**Problem:** Based on the results of the simulation, is there convincing evidence that people have a preference for the last thing they taste? Explain.

**Solution:** Yes. In the study, there were 14 people who preferred the last Kiss they tasted. This is much higher than what we would expect to happen by chance alone. In the simulation, the largest number of people choosing the last one was 9.

***Bonus #2: Cow orientation***

Do cows prefer to orient themselves in a north-south direction when grazing? A study reported in the *Arizona Daily Star* (8-26-08) looked at satellite images of 8510 cattle and found that more than 60% of them were oriented in a north-south direction rather than an east-west direction.

**Problem:** If cows have no preference for which direction they point when grazing, about how often would you expect them to be oriented in a north-south direction rather than an east-west direction?

**Solution:** If there are only two possible orientations (north-south and east-west) and cows have no preference, then we would expect about half (50%) of the cows to be oriented in north-south direction.

**Problem:** Based on the study, is there evidence that cows prefer a north-south orientation?

**Solution:** Yes, more than 60% of the cows were in the north-south direction, which is more than 50%.

**Problem:** Describe how you could use a spinner to simulate the results of this study, assuming that cows do not have a preference for a north-south orientation.

**Solution:**  Make a spinner with a 50% region for “north-south” and a 50% region for “east-west.” Spin the spinner 8510 times and count how often it lands in the “north-south” segment.

**Problem:** Here are the results of 100 trials of the simulation, assuming that 50% of the cows will prefer the north-south orientation. Describe what information is provided by the dotplot.



Simulated percent that orient north-south

**Solution:** If 50% of cows prefer to orient themselves in a north-south direction and we looked at 8510 cows, we could expect as few as 48.9% or as many as 51.2% of cows to prefer a north-south direction by random chance alone.

**Problem:** Based on the results of the simulation, is there convincing evidence that cows have a preference for a north-south orientation? Explain.

**Solution:** Yes. In the study, over 60% of cows preferred a north-south orientation. This is much higher than what we would expect to happen by chance alone. In the simulation, the largest percent of cows pointing in a north-south direction was 51.2%.

**Problem:** Researcher suspect that cows might be orienting themselves using the magnetic field of the earth. Are there other plausible explanations for why cows might orient themselves in a north-south direction?

**Solution:** Yes, it is possible that they are trying avoid sun exposure. Facing a north-south direction might help them stay cooler.

**Alternate Examples for SRIS Chapter 2**

***Page 47: Are fewer babies being born on the weekend?***

Has modern technology changed the distribution of birthdays? With more babies being delivered by planned c-section, Mrs. McDonald’s statistics class hypothesized that younger people (people born after 1993) are less likely to be born on Friday, Saturday, or Sunday compared to older people (people born before 1980). To investigate, they selected a random sample of people from both age categories and recorded the day of the week on which they were born. The results are shown in the table.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Before 1980** | **After 1993** | **Total** |
| Monday-Thursday | 45 | 49 | **94** |
| Friday-Sunday | 32 | 24 | **56** |
| **Total** | **77** | **73** | **150** |

**Problem:** Construct a graph to compare the birthdays for younger people and older people. Briefly describe what you see.

**Solution:** Here is a bar chart showing the percent born on Friday-Sunday for both groups.

Young people seem less likely to be born on Friday-Sunday. In the sample, 32.9% (24/73) of younger people were born on Friday-Sunday versus 41.6% (32/77) of older people.

**Problem:** State the hypotheses we are interested in testing.

**Solution:** : Younger people and older people are equally likely to be born on Friday-Sunday.

: Younger people are less likely to be born on Friday-Sunday than older people.

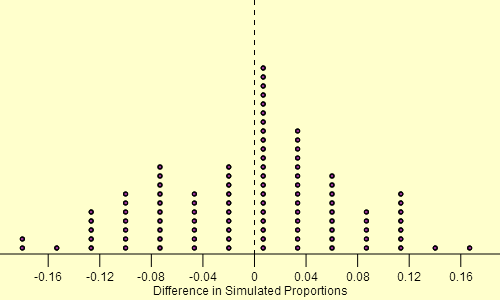
**Problem:** What is the value of the test statistic?

**Solution:** The test statistic is the difference in percent of people born on Friday-Sunday (Older – Younger) = 41.6% - 32.9% = 8.7% (0.087).

**Problem:** Describe how to simulate the distribution of the test statistic, assuming that younger and older people are equally likely to be born on Friday-Sunday.

**Solution:** Using 150 note cards, write M-Th on 94 cards and F-S on 56 cards. Shuffle the cards and divide them into two piles: one of size 77 to represent the older people and one of size 73 to represent the younger people. Calculate the percent of F-S births in each pile and subtract (Older – Younger). Record this on a dotplot and repeat many times.

**Problem:** One hundred trials of the simulation were conducted, assuming that older and younger people are equally likely to be born on Friday-Sunday. The value of the simulated test statistic for each trial was recorded on the dotplot below. Describe what information this dotplot provides.



**Solution:** The dotplot shows the possible differences in percentage of Friday-Sunday births that could arise simply by random chance, assuming that older and younger people are equally likely to be born on Friday-Sunday.

**Problem:** Using the dotplot and test statistic, estimate and interpret the *p*-value.

**Solution:** Because 14 of the 100 trials in the simulation produced a difference of 0.087 or higher, the *p*-value is approximately 14%. Assuming that older and younger people are equally likely to be born on Friday-Sunday, there is a 14% chance that we would get a difference in sample percentages of 8.7% or more by random chance.

**Problem:** Using the *p*-value, state an appropriate conclusion.

**Solution:** Because the *p*-value is large, we fail to reject the null hypothesis. We do not have convincing evidence that younger people are less likely to be born on Friday-Sunday.

***Page 52: Opinions about gun control***

Do more people favor stricter gun control than in the past? Gallup regularly asks random samples of U.S. adults their opinion on a variety of issues. In a poll of 1011 U.S. adults in January 2013, 38% agreed with the statement “I am dissatisfied with the nation’s gun laws and policies, and want them to be stricter.” In a similar poll of 1011 adults in January 2012, only 25% agreed with this statement.

**Problem:** Construct a two-way table that summarizes the results of these two polls.

**Solution:** In 2013, 38% of the 1011 people agreed with the statement, meaning that 0.38(1011) = 384 people agreed. In 2012, 25% of the 1011 people agreed, meaning that 0.25(1011) = 253 people agreed.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **2013** | **2012** | **Total** |
| Agree | 384 | 253 | **637** |
| Do not agree | 627 | 758 | **1385** |
| **Total** | **1011** | **1011** | **2022** |

**Problem:** State the hypotheses we are interested in testing.

**Solution:** : U.S. adults in 2013 and 2012 are equally likely to agree with the statement about gun control. : U.S. adults in 2013 are more likely to agree with the statement about gun control than U.S. adults in 2012.

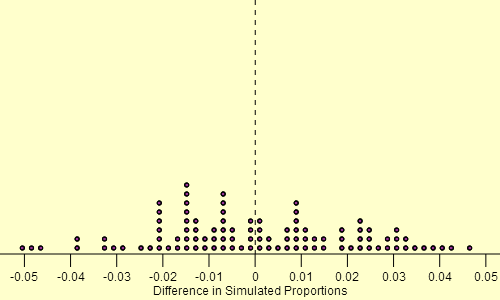
**Problem:** What is the test statistic we will use to test these hypotheses? What is the observed value of our test statistic?

**Solution:** The test statistic is the difference in percent of U.S. adults who would agree with the statement (2013 – 2012). The observed value is 38% - 25% = 13% (0.13).

**Problem:** Describe how to simulate the distribution of the test statistic, assuming that U.S. adults in 2013 and 2012 are equally likely to agree with the statement.

**Solution:** Using 2022 note cards, write Agree on 637 cards and Do Not Agree on 1385 cards. Shuffle the cards and divide them into two piles: one of size 1011 to represent the 2013 sample and one of size 1011 to represent the 2012 sample. Calculate the percent who agree in each pile and subtract (2013 – 2012). Record this on a dotplot and repeat many times.

**Problem:** One hundred trials of the simulation were conducted, assuming that U.S. adults in 2013 and 2012 are equally likely to agree with the statement. The value of the simulated test statistic for each trial was recorded on the dotplot below. Describe what information this dotplot provides.



**Solution:** The dotplot shows the possible differences in percentage who agree with the statement that could arise simply by random chance, assuming that U.S. adults in 2013 and 2012 are equally likely to agree with the statement.

**Problem:** Estimate and interpret the *p*-value.

**Solution:** Because 0 of the 100 trials in the simulation produced a difference of 0.13 or higher, the *p*-value is approximately 0%. Assuming that U.S. adults in 2013 and 2012 are equally likely to agree with the statement about gun control, there is a 0% chance that we would get a difference in sample percentages of 13% or more by random chance.

**Problem:** Based on the *p*-value, what would be an appropriate conclusion?

**Solution:** Because the *p*-value is very small, we reject the null hypothesis. We have convincing evidence that U.S. adults in 2013 are more likely to agree with the statement about gun control than U.S. adults in 2012.

**Problem:** The Sandy Hook school shooting occurred on December 14, 2012, which was between these two polls. Can we conclude that this tragedy was the cause of the increase in percent of U.S. adults who agree with the statement? Explain.

**Solution:** Not necessarily. While it is likely that many people changed their opinions about gun control following the Sandy Hook school shooting, there may have been other causes for the increase.

***Page 58: Does the wording of a question affect how people respond?***

Two statistics students decided to investigate this question by asking two different versions of a question about texting and driving. Twenty-five people at the mall were asked Version A and twenty-five different people at the mall were asked Version B. Here are the questions:

* Version A: A lot of people text and drive. Are you one of them?
* Version B: About 6000 deaths occur per year due to texting and driving. Knowing the potential consequences, do you text and drive?

The students suspected that more people would admit to texting and driving when asked Version A.

**Problem:** What are the explanatory and response variables in this experiment?

**Solution:** Explanatory: wording of the question. Response: whether or not the person admits to texting and driving.

**Problem:** What are the treatments in this experiment?

**Solution:** The treatments are using the Version A question and using the Version B question.

**Problem:** Why is it important to randomly assign which people got Version A and which people got Version B?

**Solution:** Randomly assigning the treatments helps to create two groups of 25 people that are roughly equivalent to each other. For example, if the statistics students only gave Version A to teenagers and Version B to adults, we wouldn’t be able to tell if a difference in the proportion of people who admit to texting and driving was due to their age or due to the wording of the question. Random assignment ensures that people who text and drive are equally likely to get Version A or Version B.

**Problem:** What variables are important to control (keep the same) during this experiment?

**Solution:** The people asking the questions should be the same for all 50 interviews, as the characteristics of the interviewer might also affect the responses. Also, the interviewers should conduct themselves exactly the same for each interview (approach, tone of voice, and so on).

**Problem:** State the hypotheses you are interested in testing.

**Solution:** : People at the mall are equally likely to admit to texting and driving when asked Version A and Version B. : People at the mall are more likely to admit to texting and driving when asked Version A.

**Problem:** If the students find convincing evidence in favor of the alternate hypothesis, can they conclude that the wording of the questions was the cause of the difference in response? Explain.

**Solution:** Yes, because the two groups of 25 participants should be roughly equivalent and all other variables were kept the same, except for the wording of the questions.

**Problem:** The two-way table below shows the results of this experiment. Calculate the difference in proportion of people who admit to texting (A – B). Use this difference as the test statistic.

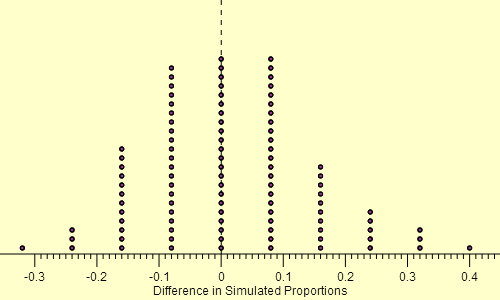
|  |  |  |  |
| --- | --- | --- | --- |
|  | **Version A** | **Version B** | **Total** |
| Admit to texting and driving | 16 | 12 | **28** |
| Do not admit to texting and driving | 9 | 13 | **22** |
| **Total** | **25** | **25** | **50** |

**Solution:** Test statistic =

**Problem:** Describe how to simulate the distribution of the test statistic, assuming that people in the mall are equally likely to admit to texting and driving with both versions of the question.

**Solution:** Using 50 note cards, write Admit on 28 cards and Do Not Admit on 22 cards. Shuffle the cards and divide them into two piles: one of size 25 to represent the people who get Version A and one of size 25 to represent people who get Version B. Calculate the percent who admit in each pile and subtract (A – B). Record this on a dotplot and repeat many times.

**Problem:** One hundred trials of the simulation were conducted, assuming that people in the mall are equally likely to admit to texting and driving with both versions of the question. The value of the simulated test statistic for each trial was recorded on the dotplot below. Estimate and interpret the *p*-value.



**Solution:** Because 19 of the 100 trials in the simulation produced a difference of 0.16 or higher, the *p*-value is approximately 19%. Assuming that people in the mall are equally likely to admit to texting and driving with both versions of the question, there is a 19% chance that we would get a difference in sample proportions of 0.16 or more by random chance.

**Problem:** Based on the *p*-value, what would be an appropriate conclusion?

**Solution:** Because the *p*-value is large, we fail to reject the null hypothesis. We do not have convincing evidence that people in the mall are more likely to admit to texting and driving when asked Version A of the question.

**Problem:** The students were surprised that they didn’t find convincing evidence that the wording of the questions made a difference. Explain one thing that the students might have done to make them more likely to find convincing evidence.

**Solution:** If the wording really makes a difference, asking more people each version of the question would help them get convincing evidence that the wording makes a difference. With relatively small group sizes, getting a difference of 0.16 is fairly likely to happen just by random chance. With larger samples, a difference of 0.16 would be much more unlikely.

**Alternate Examples for SRIS Chapter 3**

***Page 82: Is Tucson on a Windy Streak?***

The headline of an article in the *Arizona Daily Star* (May 7, 2014) stated that “Noses know spring’s been windier than usual so far.” But is the amount of wind independent from day to day? To investigate, each day was classified as a success (s) if the average wind speed was at least 6 mph and a failure (f) otherwise. Here are the results from January to April 2014 for Tucson, Arizona. Overall, there were 68 successes and 42 failures for these 110 days.

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**Problem:** Explain what it means to say that the amount of wind is independent from day to day.

**Solution:** If the amount of wind is independent from day to day, then knowing if it was windy on one day doesn’t help us predict if it will be windy the next day. In other words, it is equally likely to be windy when the previous day was windy and when the previous day was not windy.

**Problem:** State the hypotheses we are interested in testing.

**Solution:** : It is equally likely to be windy following a windy day and following a non-windy day. : It is more likely to be windy following a windy day than following a non-windy day.

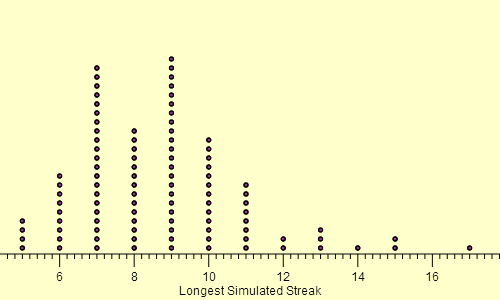
**Problem:** What is the longest observed streak of successes or failures? Use this value as the test statistic.

**Solution:** There was a streak of 15 consecutive windy days towards the end of the four-month period.

**Problem:** Describe how to simulate the distribution of the test statistic, assuming that the amount of wind is independent from day to day.

**Solution:** Using 110 note cards, label 68 with “S” to represent the 68 windy days and 42 with “F” to represent the 42 non-windy days. Shuffle the cards, lay them out in a random order, and count the longest streak of successes or failures. Repeat many times.

**Problem:** Here are the results of 100 trials of the simulation described above. Explain what information is provided by the dotplot below.



**Solution:** The dotplot shows that is possible to have a longest streak of anywhere between 4 and 17 days in a row, simply by random chance, assuming that the amount of wind is independent from day to day.

**Problem:** Estimate and interpret the *p*-value.

**Solution:** Because 3 of the 100 simulations produced a longest streak of 15 or more days in a row, the *p*-value is approximately 3%. Assuming that the amount of wind is independent from day to day, there is a 3% chance of getting a longest streak of 15 or more days in a row simply by random chance.

**Problem:** Based on the *p*-value, make an appropriate conclusion.

**Solution:** Because the *p*-value is small, we reject the null hypothesis. We have convincing evidence that it is more likely to be windy following a windy day than following a non-windy day.

***Page 87: Is Tipping Contagious?***

A cashier at Starbucks has noticed that tips seem to be streaky. That is, there are times when several customers in a row will leave tips and other times when several customers in a row won’t leave tips. She speculates that customers are influenced by the behavior of the customer in front of them and tip (or don’t tip) accordingly. Is this true? Here are the results of 50 consecutive customers, where “s” represents the 25 customers who left a tip and “f” represents the 25 customers who didn’t leave a tip.

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**Problem:** Explain what it means if the tipping outcomes of customers are independent.

**Solution:** Knowing that the previous customer left a tip doesn’t help us predict if the next customer leaves a tip. In other words, a person is equally likely to leave a tip when the previous customer left a tip and when the previous customer didn’t leave a tip.

**Problem:** State the hypotheses we are interested in testing.

**Solution:** : Customers are equally likely to tip following a tipping customer and following a non-tipping customer. : Customers are more likely to tip following a tipping customer than following a non-tipping customer.

**Problem:** How many streaks of 3 in a row were there among these 50 customers?

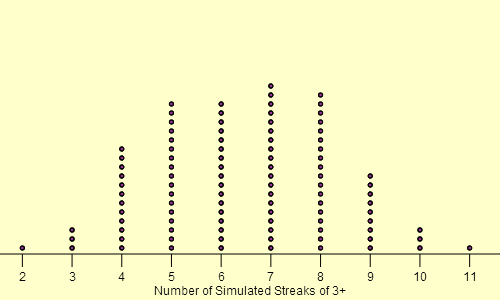
**Solution:** There were 8 streaks of at least 3 or more in a row (3 or more successes or 3 or more failures). These are underlined below:

fssssffsffffsssfsffffssssffsfsfffsfffssssfsfsfssfs

**Problem:** Describe how to simulate the distribution of the test statistic, assuming that tipping is independent from customer to customer.

**Solution:** Using 50 note cards, label 25 with “S” to represent the 25 customers who left a tip and 25 with “F” to represent the 25 customers who didn’t leave a tip. Shuffle the cards, lay them out in a random order, and count the number of streaks of 3 or more. Repeat many times.

**Problem:** Here are the results of 100 trials of the simulation described above. Explain what information is provided by the dotplot below.



**Solution:** The dotplot shows that is possible to have anywhere from 2 to 11 streaks of at least 3 in a row, simply by random chance, assuming that tipping is independent from customer to customer.

**Problem:** Estimate and interpret the *p*-value.

**Solution:** Because 31 of the 100 simulations produced 8 or more streaks of at least 3 customers in a row, the *p*-value is approximately 31%. Assuming that tipping is independent from customer to customer, there is a 31% chance of getting a 8 or more streaks of at least 3 in a row simply by random chance.

**Problem:** Based on the *p*-value, make an appropriate conclusion.

**Solution:** Because the *p*-value is large, we fail to reject the null hypothesis. We do not have convincing evidence that customers are more likely to tip following a tipping customer than following a non-tipping customer. That is, it is possible that tipping is independent from customer to customer and the streakiness we observed was just due to random chance.

***Page 93: Synchronized Traffic Lights?***

On some major roads, stoplights are synchronized so that groups of cars going near the speed limit should encounter long streaks of green lights. A student wonders if lights are synchronized on a major road near his house, so he drives north on the road for 15 lights and records the outcome of each light (“G” = green and “R” = red). Here are the results:

G G G R R R R G G G R G G G G

**Problem:** State the hypotheses that the student is interested in testing.

**Solution:** : Stoplights on this road are equally likely to be green following a green light and following a red light. : Stoplights are more likely to be green following a green light than following a red light.

**Problem:** Starting with the second outcome, classify each outcome as green after green, red after green, green after red, and red after red. Then, summarize these outcomes in a two-way table.

**Solution:** The second outcome was a green light. Because the previous light was also green, this is classified as “green after green.” Here is the two-way table summarizing all the outcomes:

|  |  |  |  |
| --- | --- | --- | --- |
|  | **After green** | **After red** | **Total** |
| **Green** | 7 | 2 | **9** |
| **Red** | 2 | 3 | **5** |
| **Total** | **9** | **5** | **14** |

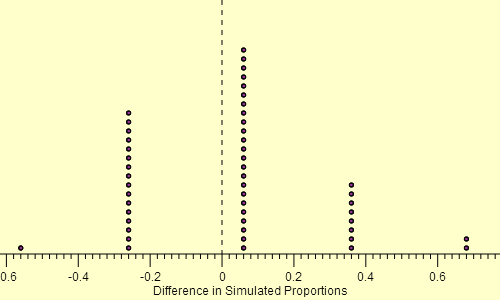
**Problem:** When the previous light was green, how often was the next light green? When the previous light was red, how often was the next light green? Use the difference in these percentages as the test statistic?

**Solution:** After a green light, 7/9 = 77.8% of the lights were green. After a red light, only 2/5 = 40% of the lights were green. The test statistic is the difference in these percentages (after green – after red): test statistic = 77.8% – 40% = 37.8%.

**Problem:** Describe how to simulate the distribution of the test statistic, assuming that lights are equally likely to be green following a green light and following a red light.

**Solution:** Using 14 note cards, label 9 with “G” to represent the 9 times the “second” light was green and 5 with “R” to represent the 5 times the “second” light was red. Shuffle the cards and divide them into two stacks, one of 9 to represent the 9 times that the “first” light was red and one of 5 to represent the 5 times that the “first” light was red. Calculate the percentage of green lights in each pile and subtract. Repeat many times.

**Problem:** Here are the results of 50 trials of the simulation described above. Use the dotplot to estimate and interpret the *p*-value.



**Solution:** Because 10 of the 50 simulations produced a difference of 0.378 or larger, the *p*-value is approximately 10/50 = 20%. Assuming that lights are equally likely to be green following a green light and following a red light, there is a 20% chance of getting a difference in percentages of 37.8% or larger, simply by random chance.

**Problem:** Is there convincing evidence that lights on this road are synchronized? Explain.

**Solution:** No. Because the *p*-value is large, we fail to reject the null hypothesis. We do not have convincing evidence that green lights are more likely following a green light than following a red light. That is, it is possible that lights are equally likely to be green following a green light and following a red light and the difference in percents was due to random chance alone.

***Page 98: Ewww…Cooties!***

A middle school principal noticed that students at her school tended to hang out in groups of mostly girls or mostly boys. Presumably, this is to prevent the spread of cooties throughout the school. As the students entered one-by-one into an assembly, the principal recorded the gender of each student so she could test the following hypotheses:

: Girls are equally likely to enter the assembly following a girl and following a boy.

: Girls are more likely to enter the assembly following a girl than following a boy.

**Problem:** Describe a Type I and a Type II error in this situation.

**Solution:** A Type I error is finding convincing evidence that girls are more likely to enter the assembly following a girl than following a boy, when in reality girls are *equally* likely to follow a girl or follow a boy.

A Type II error is not finding convincing evidence that girls are more likely to enter the assembly following a girl than following a boy, when in reality girls *are* more likely to follow a girl than follow a boy.

**Problem:** Suppose that the principal found statistically significant evidence that girls are more likely to enter the assembly following a girl than following a boy. Explain the meaning of the phrase statistically significant in this context. What does this tell you about the *p*-value?

**Solution:** If the evidence was statistically significant, it means that the principal could essentially rule out random chance as an explanation for the streaks of girls and boys that she observed. This means that the *p*-value for her test was small.

**Problem:** When the middle school principal shared these results with a teacher at a high school, the teacher decided to do a similar investigation. After collecting data at the next assembly and performing a test, the teacher obtained a *p*-value of 23%. Is this evidence statistically significant at the 10% level? Explain your answer and state an appropriate conclusion.

**Solution:** Because the *p*-value is greater than 10%, the evidence is not statistically significant. The streaks observed by the teacher could be due to random chance alone. Because of the large *p*-value, there is not convincing evidence that girls are more likely to enter the assembly following a girl than following a boy.

**Problem:** If the teacher’s conclusion was in error, which type of error did he make? Explain.

**Solution:** Type II. It is possible that the teacher did not find convincing evidence that girls are more likely to enter the assembly following a girl than following a boy, when in reality girls are more likely to follow a girl than follow a boy.

**Alternate Examples for SRIS Chapter 4**

***Page 126: More Pepperoni, Please***

Melissa and Madeline love pepperoni pizza—so much so that Melissa works at Domino’s Pizza after school. Because they are sometimes disappointed with the small number of pepperonis on their pizza, they decided to investigate the distribution of the number of pepperonis on a large pizza from Melissa’s store. Over a period of six days, Melissa counted the number of pepperonis on every fifth pizza that was made. Here are the data:

35 35 35 25 31 28 34 26 34 46 32 32 28 37 36

47 32 34 25 35 40 33 36 32 37 49 32 36 37 40

**Problem:** Construct a relative frequency histogram to display the distribution of the number of pepperonis.

**Solution:** Because the values in the distribution vary from 25 to 49, I will use classes that are 5 pepperonis wide, starting at 25. That is, the first class starts at 25 and includes values up to, but not including 30. The second class starts at 30 and includes values up to, but not including 35, and so on. Here is a frequency table and a relative frequency histogram showing this distribution.

|  |  |  |
| --- | --- | --- |
| **Pepperonis** | **Frequency**  **(number of pizzas)** | **Relative frequency**  **(percent of pizzas)** |
| 25 to <30 | 5 | 5/30 = 16.7% |
| 30 to <35 | 10 | 10/30 = 33.3% |
| 35 to <40 | 10 | 10/30 = 33.3% |
| 40 to <45 | 2 | 2/30 = 6.7% |
| 45 to <50 | 3 | 3/30 = 10.0% |
| Total | 30 | 100% |



**Problem:** Describe the shape of the histogram.

**Solution:** The distribution of number of pepperonis is unimodal and slightly skewed to the right.

**Problem:** Construct another relative frequency histogram with a different class width. How does the new histogram look the same? How does it look different?

**Solution:** Here is a relative frequency histogram with classes that are 2.5 pepperonis wide. The distribution still looks unimodal but doesn’t appear as skewed to the right. There are also some gaps (from 37.5–40 and 42.5–45) that didn’t appear in the previous histogram.



***Page 132: How heavy are the fries?***

Ryan and Brent were curious about the amount of French fries in a large order from their favorite fast food restaurant, Burger King. They went to several different Burger Kings over a series of days and ordered a total of 15 large fries. The weight of each order (in grams) is shown below:

165 163 160 159 166 152 174 166 168 173 171 168 167 170 170

**Problem:** Calculate the mean and median weight of these values.

**Solution:** To calculate the mean, add all 15 values together and divide by 15:

 grams

To calculate the median, arrange the 15 values in numerical order and identify the middle value:

152 159 160 163 165 166 166 167 168 168 170 170 171 173 174

The 8th value in this list is the median: 167 grams.

**Problem:** Graph the distribution of weight on a dotplot and label the mean and the median on the graph. Explain how the shape of the distribution helps to explain the relationship between the mean and the median.

**Solution:** The dotplot is shown below. The distribution looks slightly skewed to the left with a possible low outlier. Thus, it isn’t surprising that the mean weight is slightly smaller than the median weight.

mean median

166.1 167



Weight of large fries (in grams)

**Problem:** Calculate the range and interquartile range for the distribution of weight.

**Solution:** The range is the distance from the smallest value to the largest value:

range = 174 – 152 = 22 grams

To calculate the interquartile range (*IQR*), we need to calculate the quartiles. Using the ordered list of weights, *Q*1 is the median of the 7 values below the median and *Q*3 is the median of the 7 values above the median.

152 159 160 163 165 166 166 167 168 168 170 170 171 173 174

*Q*1 median *Q*3

*IQR* = *Q*3 – *Q*1 = 170 – 163 = 7 grams

***Page 136: How heavy are those fries, again?***

In the previous example, we calculated the quartiles and interquartile range for the distribution of weights of large fries from Burger King. Here are the data, with the quartiles labeled.

152 159 160 163 165 166 166 167 168 168 170 170 171 173 174

*Q*1 median *Q*3

**Problem:** Calculate the lower and upper boundaries for outliers in the distribution of weight. Identify any values that should be considered outliers.

**Solution:** Outliers < *Q*1 – 1.5*IQR* = 163 – 1.5(7) = 152.5 grams

Outliers > *Q*3 + 1.5*IQR* = 170 + 1.5(7) = 180.5 grams

Because 152 < 152.5, 152 grams should be considered a low outlier. Because there are no values above 180.5 grams, there are no high outliers.

**Problem:** Create a boxplot to display the distribution of weight.

**Solution:** Here is the boxplot. The lower whisker extends to the smallest value that isn’t an outlier (159 grams).



Weight of large fries (in grams)

***Page 138: Killing bacteria***

Is soap better than hand sanitizer for getting rid of unwanted bacteria? Daniel and Kate designed an experiment to find out. Using 30 identical petri dishes, they randomly selected 10 students to press their hand in a plate after washing with soap, 10 students to press their hand in a plate after using hand sanitizer, and 10 students to press their hand in a plate after using nothing. After three days of incubation, the number of bacteria colonies was counted on each plate. The distributions of number of bacteria for each treatment are summarized in the boxplots below.



Number of bacteria colonies

**Problem:** Compare the distributions of number of bacteria for the three treatments.

**Solution:** The most notable feature of the graph is that the number of bacteria was much, much higher for students who didn’t use soap or hand sanitizer. At least 75% of the students who used nothing had more bacteria than anyone who used soap or hand sanitizer. There wasn’t much difference between in the distributions for students who used soap and students who used hand sanitizer, although the values in the five number summary are slightly higher for the hand sanitizer group.

The median for the group who used nothing (53 colonies) was much larger than the median of the hand sanitizer group (6.5 colonies) and the soap group (5.5 colonies). The *IQR* was also much larger for the students who used nothing compared to the other two groups. The *IQR* for the students who used hand sanitizer was slightly larger than the *IQR* for students who used soap.

Finally, the distribution is roughly symmetric for the students who didn’t use anything, while the distributions for the students who used soap or hand sanitizer are both skewed to the right. None of the distributions has any outliers.

**SRIS Chapter 5 Alternate examples**

***Page 162: Do name-brand granola bars have more chocolate chips?***

Stephanie and Kristina wanted to know if name-brand granola bars have more chocolate chips than generic granola bars, on average. To investigate, they randomly selected 30 name-brand chewy granola bars and 30 generic chewy granola bars and carefully broke each bar apart to count the number of chocolate chips in each bar.

Here are their results:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Generic** | **Generic** | **Generic** | **Name-brand** | **Name-brand** | **Name-brand** |
| 14 | 12 | 13 | 29 | 25 | 33 |
| 6 | 9 | 13 | 26 | 41 | 27 |
| 3 | 6 | 19 | 17 | 38 | 29 |
| 14 | 15 | 6 | 14 | 26 | 27 |
| 11 | 8 | 3 | 35 | 31 | 28 |
| 13 | 10 | 8 | 29 | 30 | 37 |
| 14 | 13 | 8 | 23 | 19 | 27 |
| 15 | 12 | 17 | 35 | 26 | 25 |
| 15 | 11 | 7 | 20 | 43 | 25 |
| 14 | 6 | 11 | 27 | 20 | 27 |

**Problem:** Graph the distributions so they are easy to compare. Briefly describe what you see.

**Solution:** Here are parallel boxplots showing these data.



Both distributions look relatively symmetric. The center of the name-brand distribution is much higher than the center of the generic distribution, meaning that name-brand granola bars typically have more chips. The number of chips in name-brand granola bars is a little more variable than the number of chips in generic granola bars, meaning that the number of chips is more consistent in generic granola bars. Finally, there are three outliers in the name-brand distribution (one low outlier at 14 and two high outliers at 41 and 43).

**Problem:** If we wanted to use these sample data to determine if there is convincing evidence that name-brand granola bars have more chocolate chips than generic granola bars, on average, what hypotheses should we use? What is the value of the test statistic?

**Solution:** : The mean number of chocolate chips is the same for name-brand and generic granola bars. : The mean number of chocolate chips is greater for name-brand and generic granola bars.

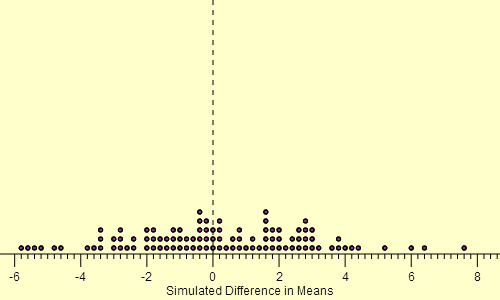
Test statistic = difference in observed means (name-brand – generic)

= 28.0 – 10.9 = 17.1.

**Problem:** Explain how to simulate the distribution of the test statistic.

**Solution:** Write each of the 60 chip counts on a note card, shuffle the cards, and distribute them at random into two piles of 30 (one for the 30 name-brand bars and one for the 30 generic bars). Then, find the mean of each pile, subtract the means (name-brand – generic), and record the difference on a dotplot. Repeat many times.

**Problem:** One-hundred trials of the simulation were conducted, assuming that the mean number of chocolate chips is the same for both types of granola bars. The difference in mean for each trial was recorded on the dotplot below. Describe what information is provided by the dotplot.



**Solution:** The dotplot shows the possible differences in sample means that could occur simply by random chance, assuming that the two types of granola bars really have the same mean number of chocolate chips.

**Problem:** Based on the results of the simulation, estimate and interpret the *p*-value.

**Solution:** Because there were no simulated differences of 17.1 or greater, the *p*-value is approximately 0/100 = 0%. Assuming that the two types of granola bars really have the same mean number of chocolate chips, there is about a 0% chance of observing a difference of sample means of 17.1 or greater by random chance.

**Problem:** Based on your *p*-value, make an appropriate conclusion. If your conclusion is in error, which type of error did you make? Explain.

**Solution:** Because the *p*-value is very small, we reject the null hypothesis. There is convincing evidence that the mean number of chocolate chips is greater in name-brand granola bars than in generic granola bars. If this conclusion is in error, it would be a Type I error: finding convincing evidence that the mean number of chips is greater in name-brand granola bars when the means are really the same.

***Page 166: Do plants prefer classical music to metal music?***

For their final project in AP Statistics, two students performed an experiment to determine if plants grow better if they listen to classical music compared to heavy metal music. Ten bean seeds were selected and each was planted in a Styrofoam cup. Half of these cups were randomly assigned to listen to metal music each night, while the other half were assigned to listen to classical music each night. The amount of growth, in millimeters, was recorded for each plant after two weeks.

**Problem:** What are the explanatory and response variables in this experiment? What are the treatments?

**Solution:** The explanatory variable is type of music. The response variable is amount of growth (mm). The treatments are listening to classical music and listening to metal music.

**Problem:** What variables would be important to control in this experiment? Why?

**Solution:** All cups should be given the same amount of water and kept in identical conditions, other than the music they are played. Otherwise the amount of growth will be more variable, making it harder to see the effect of the music.

**Problem:** Why was it important to randomly assign which seeds received the classical music treatment and which received the metal music treatment?

**Solution:** Randomly assigning the treatments will help ensure that the two groups of 5 seeds will be roughly equivalent (in terms of growing potential) at the beginning of the experiment.

**Problem:** State the hypotheses and test statistic that the students should use to address their question.

**Solution:** : Seeds grow equally well when listening to classical and metal music.

: Seeds grow better when listening to classical music than when listening to metal music.

Test statistic: Difference in mean growth (classical – metal).

Here are the results of their experiment:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Metal | 22 | 36 | 73 | 57 | 3 | = 38.2 |
| Classical | 87 | 78 | 124 | 121 | 19 | = 85.8 |



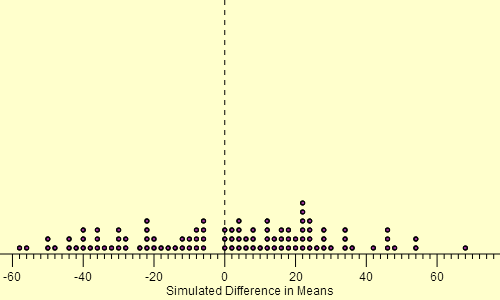
**Problem:** What is the value of the test statistic?

**Solution:** Test statistic = 85.8 – 38.2 = 47.6 mm.

**Problem:** Describe how to simulate this distribution of the test statistic.

**Solution:** Write each of the 10 growth amounts on a note card, shuffle the cards, and distribute them at random into two piles of 5 (one for the 5 beans assigned to classical music and one for the 5 beans assigned to metal music). Then, find the mean of each pile, subtract the means (classical – metal), and record the difference on a dotplot. Repeat many times.

**Problem:** One-hundred trials of the simulation were conducted, assuming that beans grow equally well when listening to classical and metal music. The difference in mean for each trial was recorded on the dotplot below. Describe what information is provided by the dotplot.



**Solution:** The dotplot shows the possible differences in sample means that could occur simply by random chance, assuming that beans grow equally well when listening to classical and metal music.

**Problem:** Based on the results of the simulation, estimate and interpret the *p*-value.

**Solution:** Because there were 4 simulated differences of 47.6 or greater, the *p*-value is approximately 4/100 = 4%. Assuming that beans grow equally well when listening to classical and metal music, there is about a 4% chance of observing a difference of sample means of 47.6 or greater by random chance.

**Problem:** Based on your *p*-value, make an appropriate conclusion.

**Solution:** Because the *p*-value is small, we reject the null hypothesis. There is convincing evidence that beans grow better when listening to classical music than when listening to metal music.

***Page 171: Do high school graduates typically earn more money than non-graduates?***

It is a common belief that students who graduate from high school earn more money than students who do not graduate from high school. Is this true? To investigate, a random sample of 371 U.S. residents aged 18 and older was selected using data from the 2000 census. The educational level and total personal income of each person was recorded. Here are dotplots showing the results for the 57 non-graduates (N) and the 314 graduates (Y).



**Problem:** Compare these distributions.

**Solution:** Both distributions are skewed to the right with several possible high outliers. The center of the distribution is higher for graduates, indicating that graduates typically have higher incomes than non-graduates in this sample. Also, the incomes for graduates are much more variable than for non-graduates.

**Problem:** The difference in median incomes (graduate – non-graduate) is $20,250 – $8000 = $12,250. Explain why it is be better to use the difference in medians as a test statistic than the difference in means.

**Solution:** Because both distributions are skewed with possible outliers, the mean income may not be a good way to estimate a typical income. Using the difference in medians is better because medians are more resistant to outliers and skewness than means.

**Problem:** Describe how to simulate the distribution of the test statistic.

**Solution:** Write each of the 371 incomes on a note card, shuffle the cards, and distribute them at random into two piles, one for the 314 for the high school grads and one of 57 for the non-grads. Then, find the median of each pile, subtract the medians (grad – non-grad), and record the difference on a dotplot. Repeat many times.

**Problem:** One-hundred trials of the simulation were conducted, assuming that the median income is the same for high school graduates and non-graduates. Use the results of the simulation below to estimate and interpret the *p*-value.



Simulated Difference in Median Income (in $1000s)

**Solution:** Because there were 0 simulated differences of $12,250 or greater, the *p*-value is approximately 0/100 = 0%. Assuming that the median income is the same for high school graduates and non-graduates, there is about a 0% chance of observing a difference of sample medians of $12,250 or greater by random chance.

**Problem:** Based on your *p*-value, make an appropriate conclusion.

**Solution:** Because the *p*-value is very small, we reject the null hypothesis. There is very convincing evidence that high school graduates have a greater median income than non-graduates.

**Problem:** If your conclusion is in error, which type of error did you make? Explain.

**Solution:** If this conclusion is in error, it would be a Type I error: finding convincing evidence that the median income for high school graduates is greater than the median income for non-graduates, when in reality the median incomes are the same.

**Problem:** Based on this study, should we conclude that having a high school diploma causes an increase in income? Explain.

**Solution:** Not necessarily. This was an observational study and not an experiment. It is possible that the people with high school diplomas are more motivated than people without high school diplomas, and their motivation is what is causing the increase in incomes.

**SRIS Chapter 6 Alternate examples**

***Page 195: Is Yawning Contagious—for Dogs?***

Many people believe that yawning is contagious. That is, seeing other people yawn makes people more likely to yawn themselves. (Just the mention of yawning probably made some of you want to yawn.) Does this apply to dogs as well? To find out, three researchers in Japan performed an experiment using 25 dogs. During different time periods, each dog was with a human that yawned or a human that made mouth movements similar to a yawn, but with no vocalization. The number of yawns was recorded for each dog in both conditions.

<Source: Romero T, Konno A, Hasegawa T (2013) Familiarity Bias and Physiological Responses in Contagious Yawning by Dogs Support Link to Empathy. PLoS ONE 8(8): e71365. doi:10.1371/journal.pone.0071365>

**Problem:** What are the explanatory and response variables in this experiment?

**Solution:** The explanatory variable is the action of the human (yawn or no yawn) and the response variable is the number of yawns made by the dog.

**Problem:** How do you know that this is paired data?

**Solution:** It is paired data because each dog was given both treatments (a human that yawned and a human that didn’t yawn).

**Problem:** Besides pairing, what other forms of control should be used?

**Solution:** The human interacting with the dog should be the same for both treatments. Also, the amount of time for each treatment and all other human behaviors should be the same for both treatments. Finally, there should be some time in-between each treatment for the dog to rest.

**Problem:** How should randomization be incorporated? Why?

**Solution:** The order of the treatments should be determined at random for each dog. Going through the experiment might make the dogs more tired (or bored!), making them more likely to yawn with the second treatment. If the dogs always had the yawning treatment second, we wouldn’t know if it was the treatment or the tiredness causing them to yawn more.

**Problem:** State the hypotheses the researcher is interested in testing.

**Solution:** : Dogs will yawn the same amount, on average, when in the presence of a yawning human and a non-yawning human. : Dogs will yawn more, on average, when in the presence of a yawning human than in the presence of a non-yawning human.

Here are the results of the experiment:

|  |  |  |  |
| --- | --- | --- | --- |
| **Dog** | **Number of Yawns**  **(human yawned)** | **Number of Yawns**  **(human didn’t yawn)** | **Difference**  **(yawn – no yawn)** |
| 1 | 2 | 0 | 2 |
| 2 | 2 | 0 | 2 |
| 3 | 0 | 1 | –1 |
| 4 | 0 | 0 | 0 |
| 5 | 1 | 1 | 0 |
| 6 | 1 | 0 | 1 |
| 7 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 |
| 10 | 0 | 1 | –1 |
| 11 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 |
| 16 | 0 | 0 | 0 |
| 17 | 0 | 0 | 0 |
| 18 | 6 | 0 | 6 |
| 19 | 1 | 1 | 0 |
| 20 | 2 | 0 | 2 |
| 21 | 5 | 0 | 5 |
| 22 | 1 | 1 | 0 |
| 23 | 2 | 0 | 2 |
| 24 | 2 | 0 | 2 |
| 25 | 0 | 0 | 0 |

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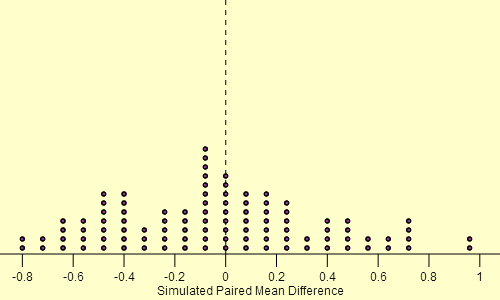
**Problem:** Calculate the mean difference (yawn – no yawn) and use this value as the test statistic. Interpret this value and describe how it provides evidence for the alternative hypothesis.

**Solution:** The mean difference is = 0.8 yawns. On average, dogs yawned 0.8 more times when in the presence of a human that yawned than when in the presence of a human that didn’t yawn. Because this value is greater than 0, there is some evidence that yawning is contagious for dogs.

**Problem:** Describe how to simulate the distribution of the test statistic, assuming that dogs will yawn the same amount, on average, when in the presence of yawning human and a non-yawning human.

**Solution:** For each dog, write each of their yawn totals on note cards. Then shuffle the two cards and randomly assign one total to the yawning treatment and one to the non-yawning treatment. Subtract the two totals (yawn – no yawn). Do this for all the dogs and find the simulated mean difference. Repeat many times.

**Problem:** Here are the results of 100 trials of the simulation. Describe what information is provided by the dotplot.



**Solution:** These are the possible mean differences that could arise by random chance, assuming that yawning isn’t contagious for dogs.

**Problem:** Use the results of the simulation to estimate and interpret the *p*-value.

**Solution:** Because 2 of the simulated mean differences are ≥ 0.8, the *p*-value is approximately 2/100 = 2%. Assuming that yawning isn’t contagious for dogs, there is a 2% chance of getting a mean difference of 0.8 or greater by random chance.

**Problem:** Based on your *p*-value, make an appropriate conclusion. If your conclusion is in error, which type of error did you make? Explain.

**Solution:** Because the *p*-value is small, we reject the null hypothesis. There is convincing evidence that dogs yawn more, on average, when in the presence of a yawning human than a non-yawning human. If this conclusion is in error, it would be a Type I error: finding convincing evidence that dogs yawn more, on average, when in the presence of a yawning human, when in reality they yawn the same amount as when around a non-yawning human.

**Problem:** How could you redesign the experiment so that it would result in unpaired data? Explain why this design isn’t as ideal as the paired design.

**Solution:** Using the same 25 dogs, randomly assign 13 to be with the yawning human and assign the remaining 12 dogs to be with the non-yawning human. Because some dogs might be more likely to yawn than others, this will create extra variability in the response, making it harder to see the effect of the treatment. Using a paired design where each dog is compared to himself/herself accounts for the variability due to the characteristics of the dogs.

***Page 200: Is the Drive-thru Faster?***

Most fast-food restaurants have a drive-thru lane. But is using the drive-thru faster than going inside the restaurant?

**Problem:** Describe one way to answer this question using unpaired data. What test statistic would you use to address this question?

**Solution:** Before heading to a fast food restaurant, I could flip a coin to determine if I will use the drive-thru lane or go inside (heads = drive-thru and tails = go inside). Each time I go, I will order the same thing and record how long it takes from the moment I enter the parking lot until I receive my food. After going to the restaurant many times, I would calculate the average time for each method and use the difference in means as my test statistic.

**Problem:** Describe one way to answer this question using paired data. What test statistic would you use to address this question?

**Solution:** Each time I go to a fast-food restaurant, I would bring a friend along. When we enter the parking lot, one of us would go inside the restaurant and the other would go through the drive-thru. To determine who goes inside, we will flip a coin (heads = I go and tails = friend goes). We will each order the same thing, record how long it takes to receive our food, and find the difference in our times. After going many times, we would calculate the mean difference in times and use this as our test statistic.

**Problem:** Describe an advantage to using paired data to answer this question.

**Solution:** Because the wait time will depend on how busy the restaurant is when we go, the amount of time it takes might vary quite a bit. If we use an unpaired design, this additional variability will make it harder to see if there is a difference in the two methods. However, using the paired design allows us to account for this variability by using both methods at the same time and comparing the two times to each other.

***Page 206: Does Generic Ice Cream Melt Faster?***

Few people enjoy melted ice cream. Being from the sunny state of Arizona, Megan and Jenna decided to test if generic vanilla ice cream melts faster than Breyers vanilla ice cream. At 30 different times during the day and night, the girls put a single scoop of each type of ice cream in the same location outside and waited for them to melt completely. The amount of time it took each type to melt (in minutes) is shown in the table below, along with the difference in times (Breyers – Generic).

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Breyers** | **Generic** | **Difference** |  | **Breyers** | **Generic** | **Difference** |
| 45 | 40 | 5 |  | 65 | 58 | 7 |
| 40 | 43 | –3 |  | 31 | 37 | –6 |
| 50 | 43 | 7 |  | 30 | 33 | –3 |
| 30 | 30 | 0 |  | 59 | 66 | –7 |
| 37 | 30 | 7 |  | 71 | 65 | 6 |
| 58 | 47 | 11 |  | 40 | 39 | 1 |
| 53 | 42 | 11 |  | 63 | 60 | 3 |
| 35 | 43 | –8 |  | 35 | 34 | 1 |
| 35 | 33 | 2 |  | 40 | 40 | 0 |
| 38 | 38 | 0 |  | 44 | 46 | –2 |
| 38 | 38 | 0 |  | 33 | 35 | –2 |
| 38 | 50 | –12 |  | 30 | 30 | 0 |
| 75 | 70 | 5 |  | 33 | 30 | 3 |
| 75 | 70 | 5 |  | 53 | 52 | 1 |
| 63 | 60 | 3 |  | 61 | 61 | 0 |

**Problem:** Explain how you know this is paired data.

**Solution:** This is paired data because both types of ice cream were placed outside at the same time in the same location.

**Problem:** Make a dotplot of the differences and describe what you see.

**Solution:** The distribution of differences is roughly symmetric with values that vary from –12 to +11. The mean difference is 1.17 minutes, meaning that the Breyers ice cream took about 1.17 minutes longer to melt, on average. In 16 trials the difference was greater than 0 and in only 8 trials was the difference less than 0, meaning that there is some evidence that the generic ice cream melts faster.



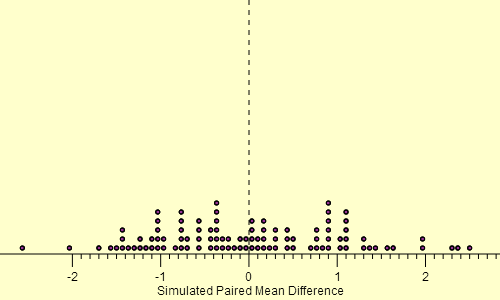
**Problem:** What hypotheses should Megan and Jenna use to answer their question?

**Solution:** : Breyers and generic ice cream take the same time to melt, on average. : Breyers ice cream takes longer to melt, on average, than generic ice cream.

**Problem:** Describe how to simulate the difference of the mean difference, assuming that Breyers and generic ice cream take the same time to melt, on average.

**Solution:** For each trial, write each of the melting times on note cards. Then shuffle the two cards and randomly assign one total to Breyers and one to generic. Subtract the two times (Breyers – generic). Do this for all the trials and find the simulated mean difference. Repeat many times.

**Problem:** Here are the results of 100 trials of the simulation. Use the results to estimate and interpret the *p*-value.



**Solution:** Because 11 of the simulated mean differences are ≥ 1.17, the *p*-value is approximately 11/100 = 11%. Assuming that Breyers and generic ice cream take the same time to melt, on average, there is an 11% chance of getting a mean difference of 1.17 or greater by random chance.

**Problem:** Based on your *p*-value, make an appropriate conclusion. If your conclusion is in error, which type of error did you make? Explain.

**Solution:** Because the *p*-value is large, we fail to reject the null hypothesis. There isn’t convincing evidence that Breyers takes longer to melt, on average, that generic ice cream. If this conclusion is in error, it would be a Type II error: not finding convincing evidence that Breyers takes longer to melt than generic ice cream, when in reality it does take longer to melt, on average.

**Problem:** Why was it a good idea to use paired data to determine if Breyers or generic ice cream melts faster?

**Solution:** The girls put the ice cream outside at different times of day and the variability in temperature will definitely increase the variability in melting time. If they didn’t use a paired design, this additional variability will make it hard to see if there is a difference between the two types of ice cream. By putting both types of ice cream out at the same time and using the difference in times, the variability in outside temperature is accounted for.

**SRIS Chapter 7 Alternate examples**

***Page 229: Chipotle: Burrito versus Bowl***

Dylan and Jaime love eating at Chipotle and wondered if there was a difference in the amount of tasty filling in a burrito and in a bowl. Over a series of weeks, these students ordered either a burrito or a bowl, with exactly the same ingredients each time. To make sure they were making a valid comparison, they put each burrito in an empty bowl before they weighed it. Likewise, they put an empty tortilla on top of each bowl before weighing it. Here are the data (in pounds):

Bowl: 1.53 1.73 1.42 1.49 1.19 1.23 1.84 1.42 1.36 1.26

Burrito: 1.45 1.38 1.34 1.45 1.34 1.22 1.63 1.51 1.27 1.61

**Problem:** Make a graph to compare these two distributions. Describe what you see.

**Solution:** The dotplots below show that both distributions are roughly symmetric and have about the same center. The weights of burritos seem more consistent than the weights of bowls.



**Problem:** The mean absolute deviation for the weight of bowls is MAD = 0.16 pounds. Interpret this value.

**Solution:** On average, the weight of a bowl is about 0.16 pounds from the mean weight.

**Problem:** Calculate the mean absolute deviation for the weights of burritos. Compare this value to the MAD for the weights of bowls.

**Solution:** Here is a table with the deviations from the mean (1.42 pounds) and the absolute values of the deviations.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Weight** | 1.45 | 1.38 | 1.34 | 1.45 | 1.34 | 1.22 | 1.63 | 1.51 | 1.27 | 1.61 | **Mean**  **= 1.42** |
| **Deviation (weight – 1.42)** | 0.03 | –0.04 | –0.08 | 0.03 | –0.08 | –0.20 | 0.21 | 0.09 | –0.15 | 0.19 |  |
| **Absolute**  **Deviation** | 0.03 | 0.04 | 0.08 | 0.03 | 0.08 | 0.20 | 0.21 | 0.09 | 0.15 | 0.19 | **MAD**  **= 0.11** |

The mean absolute deviation is MAD = 0.11 pounds. Because this is smaller than the MAD for bowls (MAD = 0.16), we can confirm that the weights of burritos seem more consistent.

***Page 232: Do you pay less at Payless?***

Mary Ann and Abigail were shopping for prom shoes and wondered how the prices at Payless compared to the prices at Famous Footwear. (Coincidentally, their statistics project proposal was due just before prom.) At each store they randomly selected 30 pairs of shoes and recorded the price of each pair. The histograms show the distribution of price for the 30 pairs of shoes from each store.

**Payless Famous Footwear**

**Problem:** Which distribution has a larger standard deviation? Explain how you know.

**Solution:** The distribution of prices at Famous Footwear has a larger standard deviation because the graphs are on the same scale and the histogram for Famous Footwear is much more spread out.

**Problem:** The standard deviation for the prices at Payless is $7.47. Interpret this value.

**Solution:** The prices of shoes at Payless are typically about $7.47 from the mean price.

**Problem:** Is the standard deviation of prices at Famous Footwear closest to $10, $15, or $30? Explain your reasoning.

**Solution:** The standard deviation is closest to $15. Most of the values are more than $10 from the mean (approximately $45), so $10 is too small to be the typical deviation. Likewise, most of the values are less than $30 from the mean, so $30 is too large to be the typical deviation. $15 seems like a typical distance from the mean, so the standard deviation is approximately $15.

**Problem:** The cheapest pair of shoes in the sample from Famous Footwear was $14.99. How does this value affect the mean and standard deviation of prices at Famous Footwear? Explain.

**Solution:** Because this pair of shoes is well below the mean, it is making the mean price smaller. Because this pair of shoes is more than 1 standard deviation from the mean, it is making the standard deviation larger.

mean – 1SD mean mean + 1SD



$14.99

**Page 236: There’s More Fish in the Bag**

Carly and Maysem randomly selected 30 bags of Original Goldfish crackers and counted the number of fish in each bag. Here are the data:

|  |  |  |
| --- | --- | --- |
| **Goldfish** | **Goldfish** | **Goldfish** |
| 333 | 335 | 349 |
| 312 | 333 | 341 |
| 337 | 327 | 320 |
| 332 | 319 | 323 |
| 329 | 330 | 325 |
| 317 | 341 | 335 |
| 342 | 323 | 324 |
| 322 | 337 | 334 |
| 337 | 330 | 330 |
| 326 | 332 | 331 |

**Problem:** Graph the distribution of the number of goldfish. Briefly describe the distribution.

**Solution:** The dotplot below shows the distribution of the number of goldfish in a bag. The distribution is roughly symmetric with a center around 330 and values that vary from 312 to 349 goldfish.



**Problem:** Calculate the mean and standard deviation for these data. Interpret the standard deviation.

**Solution:** Using a graphing calculator, I entered the 30 values into a list, performed one-variable stats, and got  = 330.2 goldfish and SD = 8.2 goldfish. The number of goldfish in a bag is typically 8.2 goldfish from the mean.

**Problem:** What percent of the values in this distribution are within one standard deviation of the mean? What percent are within two standard deviations of the mean?

**Solution:** *Within 1 SD of the mean*:  – 1 SD = 330.2 – 8.2 = 322.0 and  + 1 SD = 330.2 + 8.2 = 338.4. There are 22 values between 322 and 338.4, so 22/30 = 73% are within one standard deviation of the mean.

*Within 2 SD of the mean*:  – 2 SD = 330.2 – 2(8.2) = 313.8 and  + 2 SD = 330.2 + 2(8.2) = 346.6. There are 28 values between 313.8 and 346.6, so 28/30 = 93% are within two standard deviations of the mean.

***Page 243: Which diaper is the most consistent?***

When parents buy diapers for their children, they hope that the diapers will absorb a lot of liquid. They also hope that the amount absorbed will be consistent from diaper to diaper—otherwise there might be some unfortunate leakage!

Lily and Jake randomly selected 25 diapers of two different brands (Huggies and Pampers), making sure that all 50 diapers were for the same size child. After weighing each diaper when dry, they submerged the diaper in a tub of water for three seconds, removed the diaper, wiped off any excess water, and weighed the wet diaper. Then, they subtracted the two weights to get the total weight of water absorbed by each diaper (in grams). Here is a graph comparing the amount of water absorbed by diapers of each brand.



**Problem:** Based on the boxplots, which brand of diapers appears the most consistent?

**Solution:** The boxplot for Pampers is much less spread out, which means that Pampers appear to be more consistent than Huggies.

**Problem:** To see if there is convincing evidence that Pampers are more consistent than Huggies, what hypotheses should be test?

**Solution:** : The true standard deviation of water absorbed is the same for Pampers and Huggies. : The true standard deviation of water absorbed is the smaller for Pampers than Huggies.

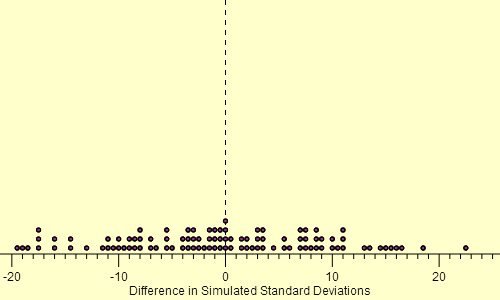
**Problem:** The observed standard deviation for Huggies is 47 grams and the observed standard deviation for Pampers is 12 grams. Calculate the test statistic, the difference in observed standard deviations.

**Solution:** Test statistic = SD of Huggies – SD of Pampers = 47 – 12 = 35 grams.

**Problem:** Explain how to simulate the distribution of the test statistic, assuming that the true standard deviation is the same for both brands.

**Solution:** For each brand, find the deviation of each diaper’s water absorption from the mean absorption of that brand (mean of Huggies = 242 , mean of Pampers = 165). Take all 50 of the deviations and write each one on a note card. Shuffle the cards and divide them into two stacks, one of 25 to represent the 25 Huggies diapers and one of 25 to represent the 25 Pampers diapers. Find the standard deviations for each stack, subtract (Huggies – Pampers), and repeat many times.

**Problem:** Here are the results of 100 trials of the simulation described above. Briefly explain what information the dotplot provides.



**Solution:** Assuming that the true standard deviations are the same for both brands, these are the possible differences in observed standard deviations that could occur by random chance alone.

**Problem:** Based on the results of the simulation, estimate and interpret the *p*-value.

**Solution:** Because there were 0 values in the simulation ≥ 35 grams, the *p*-value is approximately 0%. Assuming that the true standard deviations are the same for both brands, there is about a 0% chance of observing a difference in observed standard deviations of 35 or more by random chance alone.

**Problem:** Based on the *p*-value, make an appropriate conclusion.

**Solution:** Because the *p*-value is very small, we reject the null hypothesis. We have very convincing evidence that the true standard deviation of water absorbed is smaller for Pampers than for Huggies.

**Problem:** If your conclusion was in error, which type of error did you make? Explain.

**Solution:** A Type I error. It is possible that we found convincing evidence that the true standard deviation of water absorbed is smaller for Pampers than for Huggies, when in reality the true standard deviations are the same.

**Problem:** What is one reason a parent might consider buying Huggies? What is one reason a parent might consider buying Pampers?

**Solution:** A parent might choose Huggies because they absorb more liquid, on average, than Pampers. A parent might choose Pampers, however, if they were concerned about consistency and wanted to be able to predict more precisely when a diaper was going to fill up.

**SRIS Chapter 8 Alternate examples**

***Page 265: Which score is better?***

During her senior year, Courtney took both the SAT and ACT. Scores on the math section of the SAT vary from 200 to 800 with a mean of 514 and standard deviation of 117. Scores on the math section of the ACT vary from 1 to 36, with a mean of 21.0 and a standard deviation of 5.3.

<sources: <http://nces.ed.gov/programs/digest/d10/tables/dt10_155.asp>, <http://media.collegeboard.com/digitalServices/pdf/research/TotalGroup-2012.pdf>>

**Problem:** Courtney scored 480 on the math section of the SAT. Calculate and interpret the standardized score for this performance.

**Solution:** . Courtney’s score on the SAT math section was 0.29 standard deviations below the mean.

**Problem:** Courtney scored 18 on the math section of the ACT. Calculate and interpret the standardized score for this performance.

**Solution:** . Courtney’s score on the ACT math section was 0.57 standard deviations below the mean.

**Problem:** Relatively speaking, which of her two tests grades was better? Explain.

**Solution:** Courtney’s SAT math score was slightly better than her ACT math score. While both of her scores were below average, her SAT score was only 0.29 standard deviations below the mean while her ACT was 0.57 standard deviations below the mean.

***Page 268: Comparing GPAs***

Rebecca and her father both graduated from the same high school. When her father looked at Rebecca’s transcript, they noticed that her high school GPA (4.2) was higher than his high school GPA (3.9). After letting Rebecca gloat for a minute, he pointed out that there were no weighted grades when he went to school. To settle their argument, they called the registrar at the school and got information about the distribution of GPA in each of their graduation years. When the father graduated, the mean GPA was 2.8 with a standard deviation of 0.6. When Rebecca graduated, the mean GPA was 3.2 with a standard deviation of 0.7.

**Problem:** Calculate and interpret the *z*-score for Rebecca’s GPA.

**Solution:** . Rebecca’s GPA was 1.43 standard deviations above the mean.

**Problem:** Calculate and interpret the *z*-score for the father’s GPA.

**Solution:** . The father’s GPA was 1.83 standard deviations above the mean.

**Problem:** Relatively speaking, who had the higher GPA? Explain.

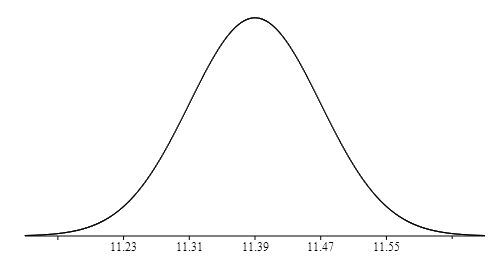
**Solution:** Rebecca’s father had the higher GPA, relatively speaking. His GPA was 1.83 standard deviations above the mean while Rebecca’s was only 1.43 standard deviations above the mean.

***Page 274-275: Weights of Oreos***

The average weight of an Oreo cookie is about 11.39 grams with a standard deviation of 0.08 grams. The distribution of Oreo cookie weight is approximately symmetric, unimodal, and bell-shaped.

**Problem:** Sketch what this distribution should look like by drawing a bell-shaped curve and labeling the mean, mean ± 1 SD, and mean ± 2 SD.

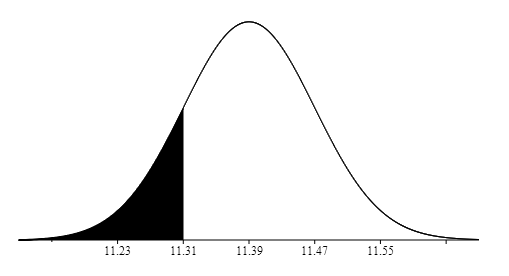
**Solution:** The mean weight is 11.39, so 11.39 will be in the middle of the distribution. To the left and right of the mean I will label 11.39 – 0.08 = 11.31 and 11.39 + 0.08 = 11.47, so that about 68% of the area will be between these two values. Then, to the left and right of these boundaries I will label 11.39 – 2(0.08) = 11.23 and 11.39 + 2(0.08) = 11.55 so that about 95% of the area will be between these two values.



Weight of Oreo Cookie (in grams)

**Problem:** About what percent of Oreo cookies will weigh less than 11.31 grams?

**Solution:** Because 68% of the weights will be between 11.31 grams and 11.47 grams, 32% of the weights will be outside of these boundaries. Because the distribution is roughly symmetric, about 16% (half of 32%) will be greater than 11.47 grams and 16% will be less than 11.31 grams.



68%

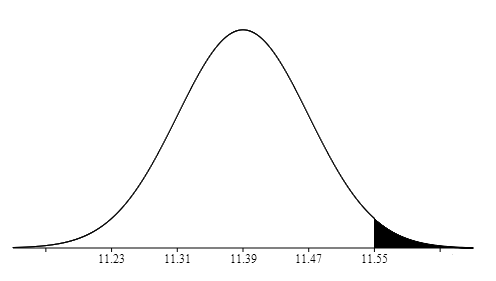
32/2 = 16%

32/2 = 16%

Weight of Oreo Cookie (in grams)

**Problem:** About what percent of Oreo cookies will weigh more than 11.55 grams?

**Solution:** Because 95% of the weights will be between 11.23 grams and 11.55 grams, 5% of the weights will be outside of these boundaries. Because the distribution is roughly symmetric, about 2.5% (half of 5%) will be less than 11.23 grams and 2.5% will be greater than 11.55 grams.



95%

5/2 = 2.5%

5/2 = 2.5%

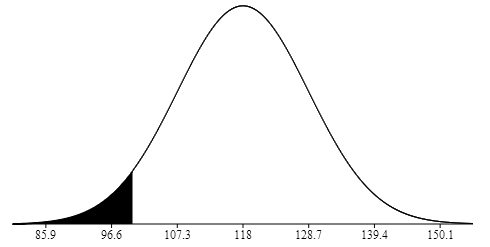
Weight of Oreo Cookie (in grams)

***Page 281: Normal Blood Pressure?***

Systolic blood pressure is the “top” number when blood pressure measurements are reported and is measured in millimeters of mercury. For 17-year-old males, the distribution of systolic blood pressure follows an approximately Normal distribution with a mean of 118.0 and standard deviation of 10.7. <Source: <http://www.nhlbi.nih.gov/guidelines/hypertension/child_tbl.pdf>>

**Problem:** About what percent of 17-year-old males have a systolic blood pressure less than 100?

**Solution:** Here is a sketch of the distribution, with the region less than 100 shaded.



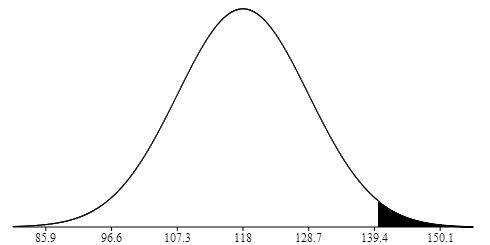
Systolic blood pressure



Using the standard normal table, the area to the left of *z* = –1.68 is 0.0465. About 4.65% of 17-year-old males have a systolic blood pressure less than 100.

**Problem:** About what percent of 17-year-old males have a systolic blood pressure greater than 140, the commonly accepted boundary for hypertension?

**Solution:** Here is a sketch of the distribution, with the region greater than 140 shaded.



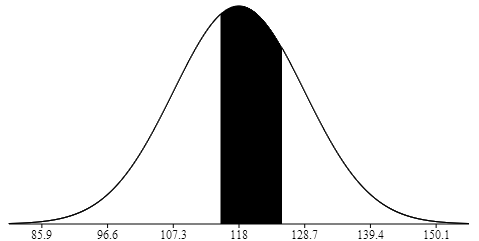
Systolic blood pressure



Using the standard normal table, the area to the left of *z* = 2.06 is 0.9803. About 100% – 98.03% = 1.97% of 17-year-old males have a systolic blood pressure greater than 140.

**Problem:** About what percent of 17-year-old males have a systolic blood pressure between 115 and 125?

**Solution:** Here is a sketch of the distribution, with the region between 115 and 125 shaded.



Systolic blood pressure

 and 

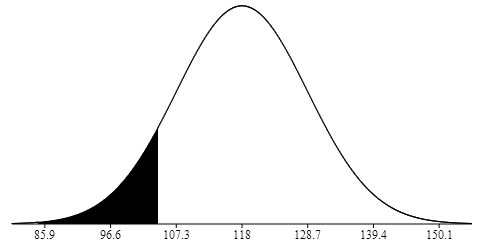
Using the standard normal table, the area to the left of *z* = –0.28 is 0.3897 and the area to the left of *z* = 0.65 is 0.7422. Thus, about 74.22% – 38.97% = 35.25% of 17-year-old males have a systolic blood pressure between 115 and 125.

***Page 284: Normal Blood Pressure?***

Systolic blood pressure is the “top” number when blood pressure measurements are reported and is measured in millimeters of mercury. For 17-year-old males, the distribution of systolic blood pressure follows an approximately Normal distribution with a mean of 118.0 and standard deviation of 10.7. <Source: <http://www.nhlbi.nih.gov/guidelines/hypertension/child_tbl.pdf>>

**Problem:** What is the 10th percentile for systolic blood pressure for 17-year-old males?

**Solution:** The 10th percentile is the boundary value that separates the lowest 10% from the highest 90% of blood pressures, as shown below.



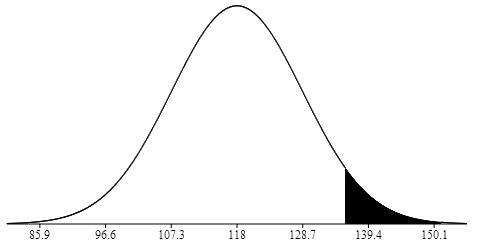
10%

Systolic blood pressure

Using the standard normal table, the area closest to 0.1000 is 0.1003 which corresponds to a *z*-score of –1.28. Solving  gives *x* = 104.3. About 10% of 17-year-old males have a systolic blood pressure less than 104.3.

**Problem:** How high must a 17-year-old male’s systolic blood pressure be to place him in the top 5%?

**Solution:** The boundary between the top 5% and the bottom 95% is shown in the diagram below.



95%

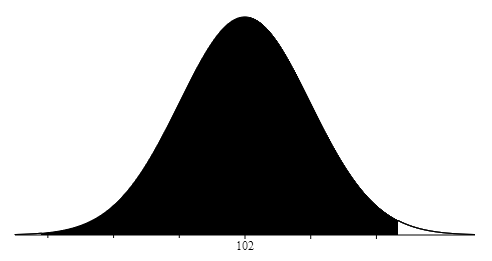
5%

Systolic blood pressure

Using the standard normal table, the area closest to 0.9500 is 0.9505 which corresponds to a *z*-score of 1.65. Solving  gives *x* = 135.7. About 5% of 17-year-old males have a systolic blood pressure greater than 135.7. [*Note*: a *z*-score of 1.64 can also be used, which results in *x* = 135.5.]

**Problem:** For 10-year-old females, the 50th percentile for systolic blood pressure is 102 and the 99th percentile is 126. Assuming that the systolic blood pressure for 10-year-old females follows an approximately Normal distribution, calculate the mean and standard deviation of the distribution.

**Solution:** The 50th percentile is the mean, so mean = 102. To find the standard deviation, we need to find the *z*-score that corresponds to an area of 0.99 as shown below.



126

1%

99%

Systolic blood pressure

Using the standard normal table, the area closest to 0.99 is 0.9901 which corresponds to a *z*-score of 2.33. Solving  gives SD = 10.3.

**SRIS Chapter 9 Alternate examples**

***Page 305: Popcorn!***

Brandon wanted to know which brand of microwave popcorn, Orville Redenbacher or Pop Secret, produces the greatest number of popped kernels, on average. He randomly selected 30 packs of each brand, microwaved each for 2 minutes, and counted the number of kernels that popped.

**Problem:** The average number of popped kernels for Pop Secret was 336.6. Explain why we cannot be 100% confident that 336.6 is the true mean for Pop Secret.

**Solution:** Brandon only used a random sample of 30 bags, so the observed mean of 336.6 is just an estimate of the true mean. To find the true mean number of popped kernels for Pop Secret, Brandon would need to pop millions of bags!

**Problem:** Using Brandon’s data, a 95% confidence interval for the true mean number of popped kernels for Pop Secret is 334.7 to 338.5. Interpret this interval.

**Solution:** We are 95% confident that the interval of plausible values from 334.7 to 338.5 contains the true mean number of popped kernels for Pop Secret microwave popcorn.

**Problem:** Explain what it means to be 95% confident in this context.

**Solution:** If Brandon were to repeat his experiment many, many times and calculate a 95% confidence interval for the true mean number of popped kernels each time, about 95% of these intervals will contain the true mean number of popped kernels for Pop Secret microwave popcorn.

**Problem:** The average number of popped kernels for Orville Redenbacher microwave popcorn was 344.5, 7.9 more than Pop Secret. A 95% confidence interval for the difference in true mean number of popped kernels (O.R. – P.S.) is 5.5 to 10.3. Does this interval provide convincing evidence that Orville Redenbacher has a greater number of popped kernels, on average? Explain.

**Solution:** Yes. Because the interval is entirely positive, there is convincing evidence that the true mean number of popped kernels is greater for Orville Redenbacher microwave popcorn than Pop Secret microwave popcorn. We are 95% confident that the true mean number of popped kernels for Orville Redenbacher is 5.5 to 10.3 greater than Pop Secret.

***Page 308: Spinning Heads***

When a fair coin is flipped, we all know that the probability the coin lands on “heads” is 0.50. However, what if a coin is spun? According to the article “Euro coin accused of unfair flipping” in the *New Scientist* (January 4, 2002), two Polish math professors and their students spun a Belgian euro coin 250 times.  A 95% confidence interval for the true proportion of heads for this spinning euro is 0.420 to 0.700.

**Problem:** Interpret the confidence interval.

**Solution:** We are 95% confident that the interval of plausible values from 0.420 to 0.700 contains the true proportion of heads when this Belgian euro coin is spun.

**Problem:** What proportion of times did the euro actually land on heads? Explain how you calculated this value.

**Solution:** The observed proportion is at the center of the confidence interval. To find this value, calculate the average of the two endpoints:

observed proportion = 

The coin landed on heads 56% of the time in their study.

**Problem:** Calculate the margin of error used in the confidence interval. Explain how you calculated this value.

**Solution:** The margin of error is the distance from the observed proportion to each of the endpoints of the confidence interval.

margin of error = 0.700 – 0.560 = 0.140

or

margin of error = 0.560 – 0.420 = 0.140

**Problem:** Interpret the margin of error in this context.

**Solution:** We expect the true proportion of heads to at most 0.140 from the observed proportion of heads (0.560).

**Problem:** One of the professors concluded that the coin was minted asymmetrically.  A representative from the Belgian mint said the result was just chance. Based on the interval, what would you conclude?

**Solution:** Because 0.50 is one of the plausible values in the interval, it is possible that the coin is “fair” and the higher proportion of heads was just due to chance.

***Page 310: Spinning the Globe***

In her first-grade social studies class, Jordan learned that 70% of the Earth’s surface was covered in water. She wondered if this was really true and asked her dad for help. To investigate, he tossed an inflatable globe to her 50 times, being careful to spin the globe each time. When she caught it, he recorded where her right index finger was pointing. In 50 tosses, her finger was pointing to water 33 times.

**Problem:** According to the data, what is our best guess for the true proportion of Earth’s surface that is covered in water?

**Solution:** Our best guess is the sample proportion, ** = 33/50 = 0.66.

**Problem:** Calculate a 95% confidence interval for the true proportion of Earth’s surface that is covered in water.

**Solution:** 🡪 🡪 0.66  0.134

🡪 0.526 to 0.794.

**Problem:** Interpret the interval in context.

**Solution:** We are 95% confident that the interval of plausible values from 0.526 to 0.794 contains the true proportion of Earth’s surface that is covered in water.

**Problem:** Is the interval consistent with the claim that 70% of Earth is covered in water? Explain.

**Solution:** Yes, because 0.70 is one of the plausible values for the true proportion included in the interval.

**Problem:** What could Jordan do to decrease the margin of error for her estimate? Explain.

**Solution:** To reduce the margin of error, Jordan and her dad should do more tosses. Because the number of tosses (*n*) is in the denominator of the margin of error, when *n* increases, the margin of error will decrease. Also, according to the law of large numbers, Jordan can expect that her estimated proportion will be closer to the true proportion when she uses more tosses to calculate her estimate.

***Page 314: Short Subs***

Abby and Raquel like to eat sub sandwiches. However, they noticed that the lengths of the “6-inch sub” sandwiches they get at their favorite restaurant seemed shorter than the advertised length. To investigate, they randomly selected 24 different times during the next month and ordered a “6-inch” sub. Here are the actual lengths of each of the 24 sandwiches (in inches), along with a dotplot:

4.50 4.75 4.75 5.00 5.00 5.00 5.50 5.50

5.50 5.50 5.50 5.50 5.75 5.75 5.75 6.00

6.00 6.00 6.00 6.00 6.50 6.75 6.75 7.00

****

**Problem:** Briefly describe the shape of the distribution. Calculate the mean and standard deviation of the sample values.

**Solution:** The shape is roughly symmetric and unimodal. The sample mean length is  = 5.68 inches with a standard deviation of *s* = 0.66 inches.

**Problem:** Calculate the 95% confidence interval for the true mean length of “6-inch” sub sandwiches from this restaurant.

**Solution:**  🡪 🡪 5.68 ± 0.27 🡪 5.41 to 5.95

**Problem:** Interpret the interval in context.

**Solution:** We are 95% confident that the interval of plausible values from 5.41 inches to 5.95 inches contains the true mean length of sub sandwiches from this restaurant.

**Problem:** Based on the interval, is there convincing evidence that the true mean length of “6-inch” sub sandwiches at this restaurant is actually less than 6 inches? Explain.

**Solution:** Yes, because all of the plausible values for the true mean length are less than 6 inches. The largest plausible value for the true mean is 5.95 inches.

**Problem:** What percentage of the 24 sub sandwiches has lengths between 5.41 and 5.95 inches? Why is this percentage so much less than 95%?

**Solution:** Only 9/24 = 37.5% of the subs had lengths between 5.41 and 5.95 inches, which is much less than 95%. However, this isn’t a contradiction. The interval describes the plausible values for the true *mean* length of the sandwiches, not the plausible values for the individual lengths of the sandwiches.

***Page 317: Is it Faster to Go Inside or Use the Drive-Thru?***

For high school students who can leave campus for lunch, this is a very important question. Ben and Maya decided to investigate this question at a local fast food restaurant. Each time they went, they randomly determine which of them would go inside and which would use the drive through. Each of them ordered the same item, paid with the same amount of cash, and recorded how long it took to wait in line, pay, and receive their item (in seconds). Here are the data:

|  |  |
| --- | --- |
| **Drive-Thru** | **Inside** |
| 325 | 170 |
| 608 | 110 |
| 90 | 52 |
| 519 | 158 |
| 216 | 66 |
| 263 | 128 |
| 559 | 81 |
| 154 | 163 |
| 449 | 64 |
| 512 | 120 |

**Problem:** Explain how you know this is paired data. Why do you think Ben and Maya used a design that produced paired data?

**Solution:** Because Ben and Maya used both methods (inside and drive-thru) each time they went, this is paired data. Using paired data was a good idea because the time of day could add a lot of variability to the wait times. If they went at an off-time, the wait time might be very short. If they went during the lunch rush, the wait time might be quite long. By doing both methods at the same time and looking at the difference in times they are able to account for this additional variability.

**Problem:** Find the difference (drive-thru – inside) in times for each of their visits and make a graph of the differences. Briefly describe what you see.

**Solution:** The differences are shown in the table and graph below.

|  |  |  |
| --- | --- | --- |
| **Drive-Thru** | **Inside** | **Difference** |
| 325 | 170 | 155 |
| 608 | 110 | 498 |
| 90 | 52 | 38 |
| 519 | 158 | 361 |
| 216 | 66 | 150 |
| 263 | 128 | 135 |
| 559 | 81 | 478 |
| 154 | 163 | –9 |
| 449 | 64 | 385 |
| 512 | 120 | 392 |



The mean difference is 258.3 seconds (over 4 minutes!) with a standard deviation of 184.9 seconds. The shape of the distribution is roughly symmetric and only one of the differences was negative. This means that in 9 of their 10 visits, it was faster to go inside than to use the drive through.

**Problem:** Calculate and interpret a 95% confidence interval for the true mean difference in times (drive-thru – inside) at this restaurant.

**Solution:** 🡪 🡪 258.3 ± 116.9 🡪 141.4 to 375.2 seconds.

We are 95% confident that the interval of plausible values from 141.4 to 375.2 seconds contains the true mean difference (drive-thru – inside) in times to wait in line, pay, and receive the order at this restaurant.

**Problem:** Does the interval provide convincing evidence that it is faster to go inside, on average? Explain.

**Solution:** Yes. Because all of the plausible values for the mean difference are greater than 0, there is convincing evidence that going inside is faster (takes less time), on average. Going inside is between 141.4 and 375.2 seconds faster, on average, compared to using the drive through.

***Page 321: Sticky Ritz?***

Sean and his sister love to eat Ritz sandwich crackers. Sean prefers the cheese variety, but his sister prefers the peanut butter variety. One day, Sean was complaining about the number of broken sandwiches (where one or both of the crackers became separated from the filling) in his box of cheese sandwiches. His sister mocked him saying that peanut butter was not only tastier, but a better binder of the crackers. To investigate, they bought several boxes and carefully counted the number of broken and unbroken sandwiches of each type. The data are summarized in the two-way table below:

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Cheese** | **Peanut Butter** | **Total** |
| Broken | 82 | 42 | **124** |
| Unbroken | 872 | 960 | **1832** |
| **Total** | **954** | **1002** | **1956** |

**Problem:** Construct a graph to compare the percent of broken cheese sandwiches and the percent of broken peanut butter sandwiches. Briefly describe what you see.

**Solution:** For cheese, there were 82/954 = 8.6% broken sandwiches. For peanut butter, there were 42/1002 = 4.2% broken sandwiches. The proportion of broken sandwiches was over twice as large for cheese than for peanut butter, supporting Sean’s sisters claim.

**Problem:** Calculate and interpret a 95% confidence interval for the difference in the true proportion of broken sandwiches (cheese – peanut butter).

**Solution:** In this problem, **= 82/954 = 0.086 and ** = 42/1002 = 0.042.

 🡪 0.044 ± 0.022

🡪 0.022 to 0.066.

We are 95% confident that the interval of plausible values from 0.022 to 0.066 contains the difference in the true proportion of broken sandwiches (cheese – peanut butter).

**Problem:** Based on the interval, is there convincing evidence that the true proportion of broken cheese sandwiches is greater than the true proportion of broken peanut butter sandwiches? Explain.

**Solution:** Yes, because all of the plausible values for the true difference are greater than 0. The true proportion of broken cheese sandwiches is between 0.022 and 0.066 greater than the true proportion of broken peanut butter sandwiches.

***Page 325: SpongeBob is in Hot Water***

Do fast-paced shows decrease a child’s ability to delay gratification? A study randomly assigned 20 4-year-olds to view a fast-paced children’s show about “an animated sponge that lives under the sea.” Another 20 4-year-olds were randomly assigned to watch a slower-paced PBS children’s show about “a typical US preschool-aged boy.” In the fast-paced show, there was a scene change every 11 seconds, on average. In the slower-paced show, there was a scene change every 34 seconds, on average.

After watching 9 minutes of the assigned program, children were asked if they preferred mini marshmallows or goldfish crackers. After choosing their snack, two plates were prepared: one with 10 pieces and one with 2 pieces. A bell was placed between them and children were told that they could have the plate with 10 if they waited for the researcher to return. Otherwise, they could ring the bell at any time and get the plate with 2 pieces. The amount of time (in seconds) it took for each child to ring the bell was recorded (330 seconds if the child waited the full time for the researcher to return). The table below shows a summary of the data:

|  |  |  |
| --- | --- | --- |
|  | **Fast-paced** | **Slower-paced** |
| Mean | 146.15 | 257.20 |
| SD | 151.29 | 132.16 |
| Sample size | 20 | 20 |

<Source: <http://pediatrics.aappublications.org/content/early/2011/09/08/peds.2010-1919>>

**Problem:** Explain why it was important for the researchers to randomly determine which children watched each show rather than letting the children choose for themselves.

**Solution:** Randomly assigning the children helps to create two groups that are roughly equivalent at the beginning of the study. Then, the only difference in their response should be due to the treatments (or random chance). If children were able to pick the program themselves, the group of kids who picked the fast-paced show would likely be different than the group of kids who picked the slower-paced show in ways that could be associated with their ability to delay gratification.

**Problem:** Calculate and interpret a 95% confidence interval for the difference in the true mean amount of time 4-year-old kids can delay gratification when watching fast-paced and slower-paced children’s shows (fast – slow).

**Solution:**  🡪 –111.05 ± 89.84

🡪 –200.89 to –21.21.

We are 95% confident that the interval of plausible values from –200.89 to –21.21 contains the difference in true mean amount of time 4-year-old kids can delay gratification (fast-paced show – slower-paced show).

**Problem:** Based on the interval, is there convincing evidence that the true mean amount of time 4-year-old kids can delay gratification is less for fast-paced shows than for slower-paced shows? Explain.

**Solution:** Yes, because all of the plausible values for the true difference are less than 0. The true mean amount of time kids can delay gratification is between 21.21 seconds and 200.89 seconds less when watching fast-paced shows.

**Problem:** Based on this study, is it reasonable to conclude that the pace of the show was the cause of the difference in means?

**Solution:** Yes. A cause-and-effect conclusion is reasonable here because the researchers performed an experiment where children were randomly assigned to watch one of the two types of shows.

**SRIS Chapter 10 Alternate examples**

***Page 351: Salads at McDonalds***

Is there a relationship between the amount of sodium (in milligrams) and amount of fat (in grams) in salads at McDonalds? Here is the data.

|  |  |  |
| --- | --- | --- |
| **Salad** | **Sodium (mg)** | **Fat (g)** |
| Southwest Salad | 150 | 4.5 |
| Southwest Salad with Grilled Chicken | 650 | 8 |
| Southwest Salad with Crispy Chicken | 820 | 21 |
| Bacon Ranch Salad | 300 | 7 |
| Bacon Ranch Salad with Grilled Chicken | 700 | 9 |
| Bacon Ranch Salad with Crispy Chicken | 870 | 22 |
| Caesar Salad | 180 | 4 |
| Caesar Salad with Grilled Chicken | 580 | 5 |
| Caesar Salad with Crispy Chicken | 740 | 18 |
| Side Salad | 10 | 0 |

<Source: <http://nutrition.mcdonalds.com/getnutrition/nutritionfacts.pdf>>

**Problem:** Make a scatterplot to show the relationship between sodium and fat, using sodium as the explanatory variable.

**Solution:** Here is the scatterplot:



**Problem:** Describe the direction of the association.

**Solution:** Overall, there is a positive association between the amount of sodium and amount of fat in salads at McDonalds. Salads with more sodium tend to have more fat.

**Problem:** Describe the form of the association.

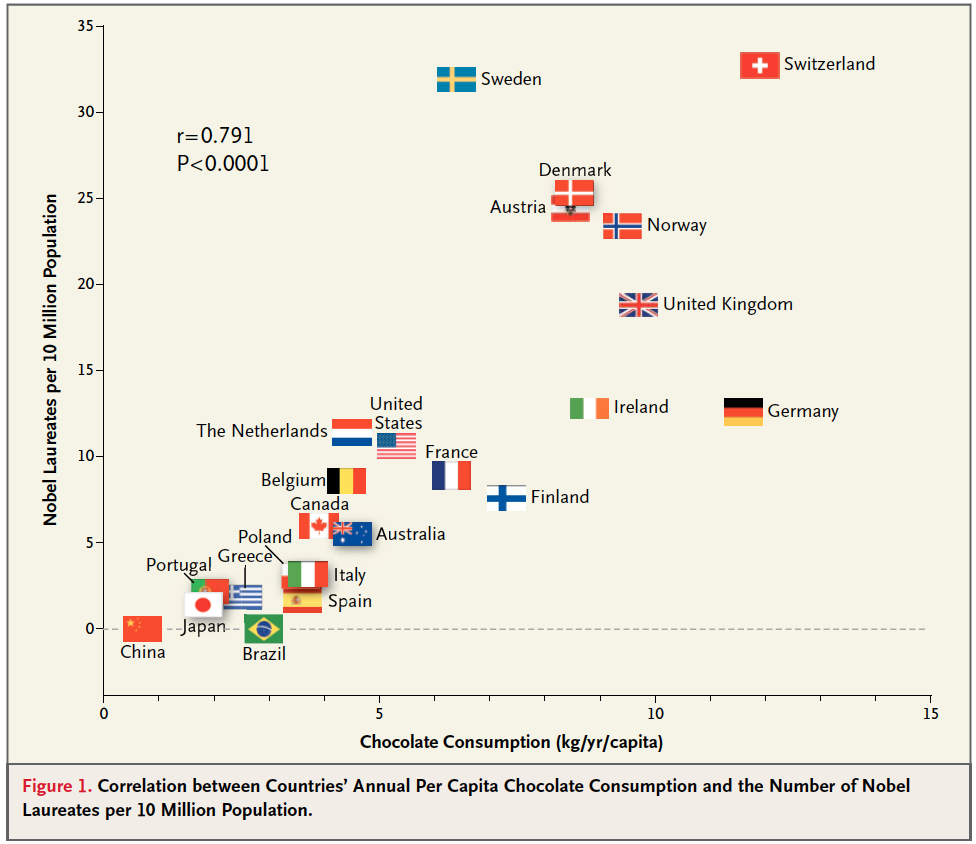
**Solution:** Because the association doesn’t follow a linear form, the association is nonlinear (curved).

**Problem:** Describe the strength of the association.

**Solution:** Because there isn’t a lot of scatter from the nonlinear form, the association is moderately strong.

***Page 356: Chocolate and Nobel Prizes***

Most people love chocolate for its great taste. But does it also make you smarter? The scatterplot below recently appeared in the New England Journal of Medicine. The explanatory variable is the chocolate consumption per person for a sample of countries. The response variable is the number of Nobel Prizes per 10 million residents of that country.



<Source: <http://www.nejm.org/doi/full/10.1056/NEJMon1211064>>

**Problem:** The correlation for these variables is *r* = 0.791. Interpret this value.

**Solution:** There is a relatively strong and positive association between chocolate consumption per person and the number of Nobel prizes per 10 million people in that country.

**Problem:** If people in the United States all started eating more chocolate, would more Nobel prizes be awarded to residents of the United States? Explain.

**Solution:** Not necessarily. Even if there is a strong correlation between two variables, we should not conclude that changes in one variable will cause changes in the other variable. There may be other variables (such as income per person) that might explain the relationship between these two variables.

***Page 359: Movie Candy***

The table below shows the amount of sugar and number of calories for 12 different candies commonly sold in movie theaters.

|  |  |  |
| --- | --- | --- |
| **Name** | **Sugar (g)** | **Calories** |
| Butterfinger Minis | 45 | 450 |
| Junior Mints | 107 | 570 |
| M&M’S | 62 | 480 |
| Milk Duds | 44 | 370 |
| Peanut M&M’S | 79 | 790 |
| Raisinettes | 60 | 420 |
| Reese’s Pieces | 61 | 580 |
| Skittles | 87 | 450 |
| Sour Patch Kids | 92 | 490 |
| SweeTarts | 136 | 680 |
| Twizzlers | 59 | 460 |
| Whoppers | 48 | 350 |

<Source: Nutrition Action, December 2009>

**Problem:** Which variable, sugar or calories should go on the *x* axis? Explain.

**Solution:** Because the amount of sugar in the candy helps explain how many calories the candy has, sugar is the explanatory variable and should be on the *x* axis.

**Problem:** Make a scatterplot of these data. Describe what you see.

**Solution:** There is a moderately strong, positive, linear relationship between the amount of sugar and number of calories in movie theater candy. The candies with more sugar typically have more calories.



**Problem:** Calculate the correlation and explain what information it provides about the relationship between points allowed and wins.

**Solution:** Using technology, *r* = 0.618. This confirms the moderately strong, positive relationship between the amount of sugar and the number of calories.

**Problem:** Explain what effect Peanut M&M’s have on the correlation. Then, temporarily remove Peanut M&M’s to confirm your answer.

**Solution:** Because the Peanut M&M’s are not in the linear pattern of the rest of the points, they are making the correlation weaker (closer to 0). When this point is removed, the correlation increases to *r* = 0.800. This confirms that the correlation is smaller when Peanut M&M’s is included.

Peanut M&M’s



**Problem:** Explain what effect SweeTarts have on the correlation. Then, temporarily remove SweeTarts to confirm your answer.

**Solution:** Because the SweeTarts are in the linear pattern of the rest of the points, they are making the correlation stronger (closer to 1). When this point is removed, the correlation decreases to *r* = 0.497. This confirms that the correlation is greater when SweeTarts is included.



SweeTarts

***Page 365: Do Older Trucks Cost Less Money?***

Most people think that as a truck gets older, its monetary value decreases. Is this true? To investigate, a random sample of 16 used Ford F-150 SuperCrew 4x4s were selected from among those on sale at Autotrader.com in 2012.

**Problem:** The scatterplot shows the relationship between number of miles driven and asking price (in dollars). Describe what you see.



**Solution:** There is a fairly strong, negative, linear relationship between the number of miles driven and asking price. Used Ford F-150s with more miles driven typically have smaller asking prices.

**Problem:** The observed correlation is *r* = –0.815. What information does this value provide?

**Solution:** Because the correlation is close to –1, this confirms that there is a strong, negative association between miles driven and asking price.

**Problem:** If you wanted to see whether the observed correlation of *r* = –0.815 provides convincing evidence that the true correlation between miles driven and price is negative, what hypotheses should you test?

**Solution:** : The true correlation between miles driven and price is 0. : The true correlation between miles driven and price is negative.

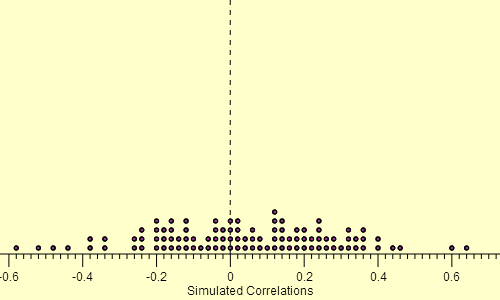
**Problem:** In this context, explain the difference between the observed correlation and the true correlation.

**Solution:** The value of the true correlation could only be known if we used every Ford F-150 in existence to calculate the correlation. The observed correlation is an estimate of the true correlation. Because the observed correlation is calculated using a random sample of 16 trucks rather than all of them, it is likely that the observed correlation isn’t equal to the true correlation.

**Problem:** Describe how to simulate the distribution of the correlation, assuming that the true correlation between miles driven and price is 0.

**Solution:** Using 16 note cards, write each of the asking prices on a card. Shuffle the cards and randomly pair them with a value for miles driven. Calculate the correlation and repeat this process many times.

**Problem:** Here are the results of 100 trials of the simulation, assuming that the true correlation is 0. Use the results to estimate and interpret the *p*-value.



**Solution:** Because 0 of the simulated correlations are ≤ –0.815, the *p*-value is approximately 0/100 = 0%. Assuming that the true correlation between miles driven and price is 0, there is a 0% chance of getting an observed correlation ≤ –0.815 by random chance alone.

**Problem:** Based on the *p*-value, make an appropriate conclusion.

**Solution:** Because the *p*-value is very small, we reject the null hypothesis. We have very convincing evidence that the true correlation between miles driven and price is negative.

***Page 371: Unemployment Rates***

Each month, the Bureau of Labor Statistics releases the unemployment rate for the United States. The table below shows the unemployment rate in January for each year from 2000 to 2014.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Year** | **Rate** |  | **Year** | **Rate** |
| 2000 | 4.0 |  | 2008 | 5.0 |
| 2001 | 4.2 |  | 2009 | 7.8 |
| 2002 | 5.7 |  | 2010 | 9.7 |
| 2003 | 5.8 |  | 2011 | 9.1 |
| 2004 | 5.7 |  | 2012 | 8.2 |
| 2005 | 5.3 |  | 2013 | 7.9 |
| 2006 | 4.7 |  | 2014 | 6.6 |
| 2007 | 4.6 |  |  |  |

<Source: <http://data.bls.gov/timeseries/LNS14000000>>

**Problem:** Construct a time plot to display any trends in the unemployment rate over the years 2000 to 2014.

**Solution:** Because the unemployment rates vary from 4.0 to 9.7, I will start the vertical scale at 3 rather than 0 to make it easier to see trends.

**Problem:** Describe any trends that you see in your time plot.

**Solution:** From about 2000 to 2002, the unemployment rate went up slowly and leveled off from 2002 to 2004. From 2004 to 2007 the rate went back down, only to rise sharply from 2007 to 2010. After 2010, the unemployment rate has been decreasing steadily, but it remains higher than any of the years from 2000 to 2008.

**Problem:** Explain why it wouldn’t be a good idea to calculate the correlation for these data.

**Solution:** It would be inappropriate to calculate the correlation because there is no linear trend in the data. Correlation only measures the strength and direction of a linear association.

***Page 374: Tornados in Texas***

Tornados can be extremely dangerous, especially powerful tornadoes with strengths of F3 or higher. Are these types of tornados becoming more common? The table below shows the number of F3 or higher tornadoes in Texas from 1997 to 2013.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Year** | **Tornados** |  | **Year** | **Tornados** |
| 1997 | 7 |  | 2006 | 2 |
| 1998 | 0 |  | 2007 | 5 |
| 1999 | 9 |  | 2008 | 0 |
| 2000 | 6 |  | 2009 | 1 |
| 2001 | 2 |  | 2010 | 3 |
| 2002 | 3 |  | 2011 | 0 |
| 2003 | 1 |  | 2012 | 2 |
| 2004 | 0 |  | 2013 | 2 |
| 2005 | 3 |  |  |  |

<Source: <http://www.ncdc.noaa.gov/>>

**Problem:** Make a time plot to show any trends in the number of tornados.

**Solution:**

**Problem:** Show how to calculate the 3-year moving average for the year 1998.

**Solution:** 3-year average =  = 5.33 tornados.

**Problem:** Find the 3-year moving average for the remainder of the years. Plot these values on the original time plot, using a different color or marking symbol.

**Solution:** The 3-year averages are marked with red squares in the time plot below.

**Problem:** Is it easier to see the overall trends using the yearly totals or the 3-year moving averages? Explain.

**Solution:** It is easier to see the overall trends with the 3-year moving averages. The yearly totals vary quite a bit from year to year, making it hard to see the overall trends. The 3-year moving averages create a smoother time plot.

**Problem:** Describe the overall trends that you see in the time plot.

**Solution:** In general, the number of F3+ tornados has been decreasing over time. In the late 1990s, there were typically around 5 or 6 tornados per year. After 2000, the number of tornados decreased for several years, which was followed by an increase to an average of around 3 in 2006. After 2006, the number of tornados has been declining to less than 2 per year, on average.

**SRIS Chapter 11 Alternate examples**

***Page 405: Lean Body Mass and Metabolic Rate***

The data table and scatterplot below show the lean body mass and metabolic rate for a sample of 5 adults. For each person, the lean body mass is the subject’s total weight in kilograms less any weight due to fat. The metabolic rate is the number of calories burned in a 24-hour period.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Mass | 33 | 43 | 40 | 55 | 49 |
| Rate | 1050 | 1120 | 1400 | 1500 | 1700 |



In this context, it makes sense to model the relationship between metabolic rate and body mass with a direct variation function in the form *y* = *kx*. After all, a person with no lean body mass should burn no calories and functions in the form *y* = *kx* always go through the point (0, 0). But what value of *k* would be best?

0

Several different values of *k* were used to predict the metabolic rate and the sum of squared residuals (SSR) was calculated for each value of *k*. Here is a scatterplot showing the relationship between SSR and *k*.



**Problem:** According to the scatterplot, when *k* = 25, SSR = 453,900. Explain what the value 453,900 represents.

**Solution:** When we use the function *y* = 25*x* to model the relationship between metabolic rate and lean body mass, the sum of squared differences between the actual metabolic rates and predicted metabolic rates is 453,900.

**Problem:** According to the scatterplot, what is the ideal value of *k* to use for predicting metabolic rate? Explain.

**Solution:** Because *k* = 31 results in the smallest sum of squared residuals, we should use the function *y* = 31*x* to predict metabolic rate from lean body mass.

**Problem:** Using the value of *k* from the previous problem, predict the metabolic rate for the subject with a lean body mass of 43 kg.

**Solution:** *y* = 31*x* 🡪 *y* = 31(43) = 1333 calories per day.

**Problem:** Calculate and interpret the residual for this subject, who actually had a metabolic rate of 1120.

**Solution:** residual = actual – predicted = 1120 – 1333 = –213. The metabolic rate for this subject was 213 calories/day less than predicted, based on his or her lean body mass.

***Page 411: Tapping on Cans***

Don’t you hate it when you open a can of soda and some of the contents spray out of the can? Two AP Statistics students, Kerry and Danielle, wanted to investigate if tapping on a can of soda would reduce the amount of soda expelled after the can has been shaken. For their experiment, they vigorously shook 40 cans of soda and randomly assigned each can to be tapped for 0 seconds, 4 seconds, 8 seconds, or 12 seconds. Then, after opening the can and cleaning up the mess, the students measured the amount of soda left in each can (in ml). Here is a scatterplot of their data. The equation of the least-squares regression line is  = 248.6 + 2.63*x*.



**Problem:** Interpret the slope of the least-squares regression line.

**Solution:** For each increase of one second in tapping time, the *predicted* amount of soda remaining will increase by 2.63 ml.

**Problem:** Does the *y* intercept have meaning in this context? Explain.

**Solution:** Yes. The *y* intercept in this context is the predicted amount of soda that remains when the can is tapped for 0 seconds. In other words, if the can is opened immediately after being shaken, we predict that there will be 248.6 mL remaining in the can.

**Problem:** Predict the amount of soda remaining for a can that was tapped for 12 seconds.

**Solution:**  = 248.6 + 2.63(12) = 280.16 ml.

**Problem:** Calculate and interpret the residual for the can that was tapped for 12 seconds and had 290 ml of soda remaining.

**Solution:** residual = actual – predicted = 290 – 280.16 = 9.84 ml. This can had 9.84 ml more soda remaining that predicted, based on how long it was tapped.

**Problem:** Would it be reasonable to predict the amount remaining if a can is tapped for 2 minutes (120 seconds)? Explain.

**Solution:** No, this would be an extrapolation. We only know that the relationship is linear for tapping times between 0 and 12 seconds. We have no idea if the relationship will look the same after 120 seconds. Furthermore, if we did make the prediction,  = 564.2 ml, which is over 200 ml more soda than was in the can to begin with (355 ml)!

***Page 420: Tipping at a Buffet***

Do customers who stay longer at buffets give larger tips? Charlotte, an APStatistics student who worked at an Asian buffet, decided to investigate this question for her second-semester project. While she was doing her job as a hostess, she obtained a random sample of receipts, which included the length of time (in minutes) the party was in the restaurant and the amount of the tip (in dollars). Here are a scatterplot which includes the least-squares regression line:  = 4.54 + 0.03*x*.



**Problem:** Interpret the slope of the least-squares regression line in context.

**Solution:** For each additional minute that a party stays at the buffet, the predicted tip increases by $0.030 (3 cents).

**Problem:** Calculate and interpret the residual for the party that stayed 65 minutes and left a tip of $8.88.

**Solution:** Predicted =  = 4.54 + 0.03(65) = $6.49.

Residual = actual – predicted = 8.88 – 6.49 = $2.39.

The tip for this party was $2.39 more than predicted, based on how long they stayed at the buffet.

**Problem:** The standard deviation of the residuals for these data is *s* = 1.78. Interpret this value in context.

**Solution:** Typically, the actual tips were about $1.78 away from the predicted tips when using the least-squares regression line with *x* = time at buffet.

***Page 424: Reaction times and Memory***

Is there a relationship between a student’s score in a memory game and his or her reaction time? A random sample of 14 high school students was selected from the CensusAtSchool data base. The scatterplot below shows the relationship between scores in a memory game and reaction times, along with the least-squares regression line. Two of the students are identified on the scatterplot as Student A and Student B.



Student B

Student A

**Problem:** What effect does Student B have on the standard deviation of the residuals? Explain.

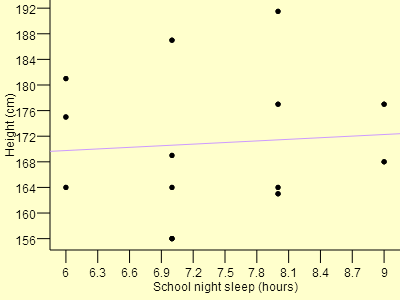
**Solution:** Because Student B has a very large residual compared to most of the other students, Students B is making the standard deviation of the residuals larger.

**Problem:** Which student, A or B, has a greater influence on the equation of the least-squares regression line? Explain.

**Solution:** Even though Student A has a smaller residual, this student has more influence on the equation of the least-squares regression line. Points that have unusually small (or large) values of the explanatory variable are much more influential than points that have more typical values of the explanatory variable. If Student A were removed, the slope of the least-squares regression line would become much steeper. However, if Student B were removed, there would be very little change to the equation of the least-squares regression line.

***Page 429: Does getting more sleep make you taller?***

Parents have been known to tell their young children that getting lots of sleep will make them taller. Is this true? A random sample of 14 high school students was selected to investigate the relationship between height (in cm) and the typical number of hours of sleep on school nights. These data are shown on the scatterplot below. The equation of the least-squares regression line is  = 164.7 + 0.835*x*.



**Problem:** Interpret the slope of the least-squares regression line in this context.

**Solution:** For each increase of 1 hour of sleep on school nights, the predicted height increases by 0.835 cm.

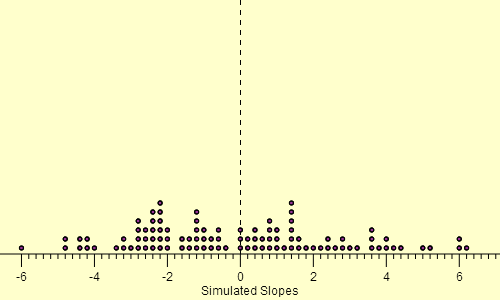
**Problem:** Using the observed slope as the test statistic, state the hypotheses we are interested in testing.

**Solution:** : The true slope of the least-squares regression line using *x* = sleep hours on a school night and *y* = height is 0. : The true slope of the least-squares regression line using *x* = sleep hours on a school night and *y* = height is positive.

**Problem:** Describe how to simulate the distribution of the slope, assuming that the true slope is 0.

**Solution:** Write the heights on note cards, shuffle the cards, randomly pair a height with a sleep time, and calculate the simulated slope. Repeat many times and record the results on a dotplot.

**Problem:** Here are the results of 100 trials of the simulation. Use the results to estimate the *p*-value, interpret the *p*-value, and make an appropriate conclusion.



**Solution:** Because 35 of the 100 simulated slopes are greater than or equal to 0.835, the *p*-value is approximately 35%. *Interpretation:* If the true slope of the least-squares regression line using *x* = sleep hours on a school night and *y* = height is 0, there is a 35% chance of getting an observed slope of 0.835 or greater by random chance. *Conclusion:* Because the *p*-value is large, we fail to reject the null hypothesis. We do not have convincing evidence that students who get more sleep on school nights tend to be taller.

**Problem:** If your conclusion was in error, which type of error did you commit? Explain.

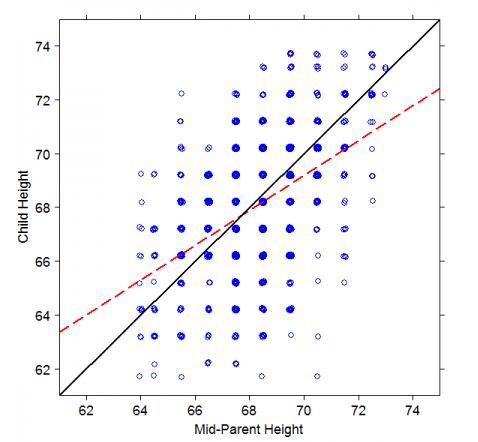
**Solution:** Type II. It is possible that we didn’t find convincing evidence that students who get more sleep on school nights tend to be taller, when in fact students who get more sleep do tend to be taller.

***Page 435: Heights of Parents and Children***

Sir Francis Galton is generally regarded as the first person to identify the concept of regression to the mean. In one of his most famous demonstrations, he made a scatterplot showing the relationship between the heights of parents and their adult children. The values on the horizontal axis represent the average of the two parent heights (called mid-parent height). The values on the vertical axis are the heights of their adult children, with female heights multiplied by 1.08 to put them on the same scale as male heights.

The scatterplot below is a re-creation of Galton’s scatterplot, with each blue circle representing one of the 930 children in the study. Because there were multiple observations at some points on the scatterplot, these circles were shifted by a small amount to make the multiple observations more obvious. The solid black line is the line *y* = *x*. The dotted red line is the least-squares regression line.

<Source: <http://www.select-statistics.co.uk/article/blog-post/regression-to-the-mean-as-relevant-today-as-it-was-in-the-1900s>>



**Problem:** What can you say about the heights of children whose parents had a mid-parent height greater than 72 inches (6 feet)?

**Solution:** When parents had a mid-parent height of greater than 72 inches, their children were all relatively tall (at least 68 inches). However, most of these children were shorter than their mid-parent height (below the black line).

**Problem:** What can you say about the heights of children whose parents had a mid-parent height less than 65 inches (5 foot 5 inches)?

**Solution:** When parents had a mid-parent height of less than 65 inches, their children were all relatively short (at most 69 inches). However, most of these children were taller than their mid-parent height (above the black line).

**Problem:** How does the equation of the least-squares regression line illustrate regression to the mean?

**Solution:** For mid-parent heights that are above average, the least-squares regression line predicts heights that are taller than average, but less tall than the mid-parent height (closer to the mean height of all children). For mid-parent heights that are below average, the least-squares regression line predicts heights that are shorter than average, but less short than the mid-parent height (closer to the mean height of all children).

**SRIS Chapter 12 Alternate examples**

***Page 463: Movie Candy***

In Chapter 10, we observed a positive relationship between sugar and calories in candy typically sold at movie theaters. We can use multiple regression to predict *y* = calories using *x*1 = sugar (in grams) and *x*2 = saturated fat (in grams). Here is the model:

 = 125.7 + 3.8*x*1 + 11.7*x*2

*s* = 60.2

**Problem:** Whoppers have 350 calories, 48 grams of sugar, and 13 grams of saturated fat. Calculate and interpret their residual.

**Solution:** The predicted number of calories for Whoppers is:

 = 125.7 + 3.8(48) + 11.7(13) = 460.2 calories.

The residual is: *y* –  = 350 – 460.2 = –110.2 calories.

Whoppers have 110.2 fewer calories than expected, based on the amount of sugar and saturated fat they contain.

**Problem:** Interpret the coefficients of *x*1 and *x*2 in the context of this problem.

**Solution:** For each increase of 1 gram of sugar, the predicted number of calories increases by 3.8, assuming that the amount of saturated fat doesn’t change.

For each increase of 1 gram of saturated fat, the predicted number of calories increases by 11.7, assuming that the amount of sugar doesn’t change.

**Problem:** Interpret the value *s* = 60.2 for this model.

**Solution:** When using the amount of sugar and saturated fat to predict the number of calories in movie candy, our predictions will typically be off by 60.2 calories.

**Page 467: Are vegetarians taller?**

In a random sample of 193 high school students, vegetarians were about 1.3 cm shorter than non-vegetarians. But most of the vegetarians were female, and females tend to be shorter than males to begin with. Let’s use multiple regression to estimate the effect of being a vegetarian after accounting for age and gender. Here is the model, where *y* = height (in cm), *x*1 = age (in years), *x*2 = gender (1 = male, 0 = female), and *x*3 = vegetarian (1 = yes, 0 = no).

 = 158.78 + 0.26*x*1 + 15.08*x*2 + 4.07*x*3

*s* = 9.35

**Problem:** Interpret the coefficient of *x*1.

**Solution:** For people of the same gender and vegetarianism status, each increase of 1 year in age corresponds to an increase in the predicted height of 0.26 cm.

**Problem:** Interpret the coefficient of *x*2.

**Solution:** If you are a male, your predicted height is 15.08 cm greater than if you are a female, assuming that your age and vegetarianism status don’t change.

**Problem:** Interpret the coefficient of *x*3.

**Solution:** If you are a vegetarian, your predicted height is 4.07 cm greater than if you are not a vegetarian, assuming that your age and gender don’t change. This means that after accounting for age and gender, vegetarians tend to be about 4 cm taller than non-vegetarians.

**Problem:** Calculate and interpret the residual of the student who is 160 cm tall, 18 years old, female, and a vegetarian.

**Solution:** Predicted height =  = 158.78 + 0.26(18) + 15.08(0) + 4.07(1) = 167.53 cm.

Residual = *y* –  = 160 – 167.53 = –7.53 cm.

This student was 7.53 cm shorter than expected, based on her age, gender, and vegetarianism.

**Problem:** Interpret the value of *s*.

**Solution:** When using age, gender, and vegetarianism to predict height for high school students, we will typically be off by about 9.35 cm.

**Problem:** If a student decides to become a vegetarian, can they expect to grow another 4 cm? Explain.

**Solution:** No, even though vegetarians tend to be about 4 cm taller than non-vegetarians, after accounting for age and gender, this is only an association. We cannot conclude that being a vegetarian *causes* students to grow taller unless we did an experiment where we randomly assigned children to be vegetarians (or not) and then compared their heights.

***Page 472: Building a Model to Predict Price of Tablets***

In January 2013, *Consumer Reports* magazine included information and review of tablets, such as Apple’s iPad. The prices of tablets vary quite a bit, based on factors such as screen size and battery life. Here is a model, where *y* = price (in dollars), *x*1 = screen size (diagonal, in inches) and *x*2 = battery life (in hours).

 = –5.83 + 31.94*x*1 + 31.44*x*2

*s* = 112.19

**Problem:** Do tablets with bigger screens tend to cost more, after accounting for battery life? Explain how you know.

**Solution:** Yes, because the coefficient of *x*1 is positive. For each increase of 1 diagonal inch in screen size, the predicted price increases by $31.94, assuming that the battery life remains the same.

**Problem:** Do tablets with longer battery life tend to cost more, after accounting for screen size? Explain how you know.

**Solution:** Yes, because the coefficient of *x*2 is positive. For each increase of 1 hour of battery life, the predicted price increases by $31.44, assuming that the screen size remains the same.

**Problem:** Interpret the value of *s*.

**Solution:** When using screen size and battery life to predict the price of a tablet, we will typically be off by about $112.19.

Let’s add a third explanatory variable to the model: *x*3 = weight (in pounds). All other things being equal, it is reasonable to think that a heavier tablet would be less desirable and cost less money. Here is the model:

 = –78.82 + 58.76*x*1 + 26.16*x*2 – 94.36*x*3

*s* = 113.97

**Problem:** Does adding the third variable (weight) make the model better or worse? Explain how you know.

**Solution:** Worse, because the value of *s* went up when we added weight to the model. It would be better to leave weight out of the model and use the simpler model with just screen size and battery life.

**SRIS Chapter 13 Alternate examples**

***Page 497: Growth of AP Statistics***

The AP Statistics exam was first given in 1997. The scatterplot below shows the growth in the number of AP Statistics exam from 1997 to 2013.



**Problem:** Describe the association you see in the scatterplot.

**Solution:** The association between the number of exams and year appears to be very strong, positive, and linear.

A least-squares regression line was calculated using *x* = year and *y* = number of exams. The equation of the least-squares regression line is  = –19,724,000 + 9877.91*x*.

**Problem:** In 1997, there were 7667 AP Statistics exams given. Calculate and interpret the residual for 1997.

**Solution:** Predicted:  = –19,724,000 + 9877.91(1997) = 2186.27 exams

Residual = *y* –  = 7667 – 2186.27 = 5480.73 exams.

In 1997, there were 5480.73 more exams given than predicted, based on the least-squares regression line using *x* = year.

Using the equation of the least-squares regression line, the following residual plot was constructed.



**Problem:** Based on the residual plot, is a linear model appropriate for this association? Explain.

**Solution:** No.Because there is a leftover curved pattern in the residual plot, the linear model is not the best model. Even though the original scatterplot looks linear, there is a slight curve that is much easier to see in the residual plot.

**Problem:** If you were to use the linear model to predict the number of AP exams in 2014, would you expect your prediction to be too low, too high, or just about right?

**Solution:** Because the pattern of the residuals suggests that the residual for 2014 will be positive, our prediction will likely be too small. If actual – predicted > 0, then actual > predicted.

***Page 504: Diamonds are Forever…and Expensive!***

The table below shows the price (in dollars) and carat weight for 15 randomly selected round, clear, internally flawless diamonds with excellent cuts.

|  |  |  |  |
| --- | --- | --- | --- |
| **Carat** | **Price** | **Carat** | **Price** |
| 0.50 | 4070 | 1.04 | 24,679 |
| 3.61 | 365,700 | 1.20 | 24,855 |
| 1.21 | 30,315 | 1.10 | 25,127 |
| 1.74 | 57,282 | 1.08 | 24,026 |
| 0.31 | 1404 | 1.61 | 47,126 |
| 3.56 | 329,613 | 1.29 | 32,315 |
| 1.51 | 45,928 | 1.13 | 25,075 |
| 3.01 | 229,509 |  | |

<Source: [http://www.lumeradiamonds.com/diamonds/results?price=1082-1038045&carat=0.30-16.03&shapes=B&cut=EX&clarity=FL,IF&color=D,E,F#](http://www.lumeradiamonds.com/diamonds/results?price=1082-1038045&carat=0.30-16.03&shapes=B&cut=EX&clarity=FL,IF&color=D,E,F)>

**Problem:** Make a scatterplot to show the relationship between price and carat. Describe the association.

**Solution:** The scatterplot is shown below. There is a strong, positive, nonlinear association between the price of a diamond and the carat weight.



**Problem:** Calculate the quadratic model for this association and graph it on your scatterplot.

**Solution:** Using my graphing calculator, the quadratic model is:

 = 34,473*x*2 – 30,698*x* + 13,151



**Problem:** Calculate and interpret the residual for the diamond that weighed 3.61 carats and cost $365,700.

**Solution:** Predicted =  = 34,473(3.61)2 – 30,698(3.61) + 13,151 = $351,587.

Residual = 365,700 – 351,587 = $14,113.

This diamond cost $14,113 more than expected, based on its carat weight.

**Problem:** Sketch the residual plot for this model and use it to assess the appropriateness of the model.

**Solution:** Because the residual plot shows no leftover patterns, the quadratic model is appropriate for this association.



***Page 511: Eating M&M’S***

A student opened a bag of M&M’S, dumped them out, and ate all the ones with the M on top. When he finished, he put the remaining 30 M&M’S back in the bag and repeated the same process over and over until all the M&M’s were gone. Here is scatterplot showing the number of M&M’S remaining at the end of each “course.”

****

**Problem:** Describe the association between the number remaining and the course number.

**Solution:** There is a strong, negative, nonlinear association between the number remaining and the course number. As the courses go by, the number remaining gets closer and closer to 0.

**Problem:** The graph of the exponential model  = 57.93(0.506)*x* is shown on the scatterplot below, along with the corresponding residual plot. Use these graphs to discuss if the exponential model is appropriate.

**Solution:** On the scatterplot, the exponential model seems to follow the same form as the association. This is confirmed by the residual plot, which shows no leftover pattern. Thus, the exponential model seems appropriate to model this association.

**Problem:** Use the exponential model to calculate and interpret the residual for the third course, when there were 10 M&M’S remaining.

**Solution:** Predicted =  = 57.93(0.506)*3* = 7.51 M&M’S.

Residual = *y* –  = 10 – 7.51 = 2.49 M&M’S.

There were 2.49 more M&M’S remaining after the third course than expected, according to the exponential model.

**Problem:** Interpret the base of the exponential model.

**Solution:** The base of the exponential model is *b* = 0.506, so the percent decrease is (1 – 0.506)100% = 49.4%. For each additional course, the predicted number of M&M’S remaining decreases by 49.4%. This seems quite reasonable as we would expect about half (50%) of the M&M’S to land with the M side up each time.

**Problem:** Interpret the coefficient of the exponential model.

**Solution:** The coefficient of the exponential model is *a* = 57.93. Because *x* = 0 represents the initial amount of M&M’S before the first course was eaten, we predict that there were about 58 M&M’S in the bag to begin with.

***Page 519: Older Home Owners***

Are older people more likely to own their home? This might seem reasonable because as people get older, they typically have higher incomes and have had longer to save the money needed for a home. A random sample of 387 U.S. residents age 18 and older was selected from the 2000 census. Using these data, the following logistic model was calculated using age (in years) as the explanatory variable and whether or not they own their home (1 = own or paying off a home loan, 0 = rent/other) as the response variable:



**Problem:** Does the equation of the logistic model reveal a positive or negative association between age and owning a home? Explain how you know.

**Solution:** Because the coefficient of *x* is positive, there is a positive association between age and owning a home. The older a person gets, the more likely he or she is to own a home.

**Problem:** Calculate and interpret the value of ** for a 40-year-old.

**Solution:** = 0.635.

I predict that about 63.5% of 40-year-olds will own their own home. In other words, a randomly selected 40-year-old has a 63.5% chance of owning his or her home.

**Problem:** Calculate the value of ** when *x* = 20, 60, 80, and 100. Use these values and the value from the previous problem to sketch the graph of the logistic model.

**Solution:** The table below shows the values of ** for *x* = 20, 40, 60, 80, and 100. These values were plotted on a scatterplot and connected with a smooth curve that stays above 0 and below 1.



|  |  |
| --- | --- |
| Age |  |
| 20 | 0.477 |
| 40 | 0.635 |
| 60 | 0.769 |
| 80 | 0.865 |
| 100 | 0.924 |

***Page 526: Who will Enter the Hall of Fives?***

Students who earn a 5 (the highest score) on the AP Statistics exam in Mr. Tabor’s class have their names engraved on the “Hall of Fives.” Which variable is a better predictor of getting a 5, score on the midterm exam or score on the final exam? The scatterplots below show data for students in 2013 and the corresponding logistic model for the predicted probability of scoring a 5.

**Problem:** Briefly describe the direction of each association.

**Solution:** Both associations are positive—the greater the midterm or final exam score, the greater the chance of earning a 5 on the AP Statistics exam.

**Problem:** Which variable has a stronger association with earning a 5, score on the midterm or score on the final? Explain.

**Solution:** Because the logistic curve using final exam score is steeper than the logistic curve using midterm exam score, final exam score has a stronger relationship with earning a 5 on the AP Statistics exam.

**Problem:** If a student wanted a 50% chance of earning a 5 on the AP Statistics exam, what score on the final exam do they need? Explain.

**Solution:** To determine the score needed on the final that corresponds to a 50% chance of earning a 5, draw a horizontal line from Pass5 = 0.50. Then, draw a vertical line to the *x* axis from where the horizontal line intersects the logistic curve. Because the vertical line intersects the *x* axis around 136, students that earn 136 points on the final exam have a 50% chance of earning a 5 on the AP Statistics exam.



**SRIS Chapter 14 Alternate examples**

***Page 547: Picking an Outfit***

Each day, a quirky math teacher picks his clothes at random. He has 10 shirts, 4 pairs of pants, 2 belts, 10 pairs of socks, and 3 pairs of shoes.

**Problem:** How many different outfits can the math teacher choose?

**Solution:** Using the fundamental counting principle, I can multiply the number of options the teacher has in each category to get the total number of possible outfits:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Shirts | Pants | Belts | Socks | Shoes | Total |
| # of Options | 10 | 4 | 2 | 10 | 3 | 10 × 4 × 2 × 10 × 3 = 2400 |

There are 2400 different possible outfits the math teacher can choose.

***Page 549: Hanging Art***

Hana’s parents want to hang some of the 12 paintings that she made in her first grade class. They will hang them on a single curtain wire that stretches along a wall, as shown below.

**Problem:** If Hana’s parents want to hang all 12 of her paintings, how many different orders are possible?

**Solution:** The table summarizes the number of choices Hana’s parents have for each position.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Position | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| # of options | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |

Using the fundamental counting principle:

12 × 11 × 10 × 9 × 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1 = 12! = 479,001,600

There are 479,001,600 different possible orders.

**Problem:** If Hana’s parents only have room for 8 paintings, how many different orders are possible?

**Solution:** The table summarizes the number of choices Hana’s parents have for each position.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Position | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| # of options | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 |

Using the fundamental counting principle:

12 × 11 × 10 × 9 × 8 × 7 × 6 × 5 = 19,958,400

There are 19,958,400 different possible orders.

I can also find this by calculating 12P8 = 

***Page 552: Choosing Students***

At the end of each week, a generous teacher gives some candy to a few randomly selected students. There are 30 students in this teacher’s class, with 16 girls and 14 boys.

**Problem:** If the teacher wanted to give candy to 6 of the students, how many possible groups of 6 students can she choose?

**Solution:** Because all students chosen will receive the candy, the order that the students are chosen in doesn’t matter. Thus, we want to know the number of combinations when choosing 6 students from a total of 30 choices:

30C6 =  = 593,775

There are 593,775 different groups of 6 students that can be chosen.

**Problem:** To avoid appearing to favor one gender, the teacher will randomly choose 3 of the 16 girls and 3 of the 14 boys. How many possible groups of 6 can she choose? How does this compare to the previous problem?

**Solution:** Because the order that the girls are chosen doesn’t matter, this is a combinations problem. The number of ways to choose 3 girls from 16 choices is:

16C3 =  = 560.

Likewise, the number of ways to choose 3 boys from 14 choices is:

14C3 =  = 364.

Finally, using the fundamental counting principle, the total number of groups of 6 that can be chosen is: 560 × 364 = 203,840. This is a smaller number of possible groups than in the previous problem because the teacher has put a restriction on the makeup of the group (it must be 3 girls and 3 boys).

***Page 558: Attending a Convention***

A high school community service club is allowed to send 4 students to a national convention. The club advisor decides to select 4 of the club members at random to attend. The club has 50 members, with 21 seniors, 11 juniors, 6 sophomores, and 12 freshmen.

**Problem:** How many ways are there to select 4 club members to attend the convention?

**Solution:** Because the order of selections doesn’t matter, this is a combinations problem. The number of possible ways is:

50C4 =  = 230,300.

**Problem:** Some club members were upset when there were 3 seniors chosen for the trip. How many different ways are there to choose 3 seniors and 1 non-senior?

**Solution:** There are 21C3 ways to choose 3 seniors and 29C1 ways to choose 1 non-senior. Thus, there are (21C3)(29C1) = (1330)(29) = 38,570 ways to choose 3 seniors and 1 non-senior.

**Problem:** If the 4 students are chosen at random, what is the probability that there will be 3 seniors and 1 non-senior?

**Solution:** *P*(3 seniors and 1 non-senior) =  = 0.167.

**Problem:** Interpret the probability that you calculated in the previous part.

**Solution:** If the club advisor randomly selected a group of 4 students many, many times, about 16.7% of the time the advisor would select a group that included 3 seniors and 1 non-senior.

***Page 560: Attending a Convention, part 2***

A high school community service club is allowed to send 4 students to a national convention. The club advisor decides to select 4 of the club members at random to attend. The club has 50 members, with 21 seniors, 11 juniors, 6 sophomores, and 12 freshmen. Some club members were upset when there were 3 seniors chosen for the trip.

**Problem:** How many different ways are there to choose *at least* 3 seniors?

**Solution:** Choosing at least 3 seniors means that the advisor chooses 3 seniors and 1 non-senior OR 4 seniors and 0 non-seniors.

From the previous problem, *P*(3 seniors and 1 non-senior) = .

Using the same reasoning, *P*(4 seniors and 0 non-senior) = .

Thus, *P*(at least 3 seniors) =  = 0.193.

**Problem:** Interpret the probability that you calculated in the previous problem.

**Solution:** If the club advisor randomly selected a group of 4 students many, many times, about 19.3% of the time the advisor would select a group that included at least 3 seniors.

**Problem:** If there was also a state convention, what is the probability that the advisor selects at least 3 seniors to go to each convention? Would this make you suspicious that the selection wasn’t at random? Explain.

**Solution:** *P*(at least 3 seniors for both conventions)

= *P*(at least 3 seniors) × *P*(at least 3 seniors) = 0.193 × 0.193 = 0.037.

Because this probability is small, I would be suspicious. It would be unlikely to select at least 3 seniors for both conventions by random chance alone.

**Problem:** What is the probability that the teacher does *not* select at least 3 seniors for both conventions?

**Solution:** *P*(does not select at least 3 seniors for both conventions)

= 1 – *P*(selects at least 3 seniors for both conventions)

= 1 – 0.037 = 0.963.

***Page 564: Rolling a Die***

When a single die is rolled, the probability of getting a six is 1/6.

**Problem:** If you roll a die three times, what is the probability that you get no sixes?

**Solution:** *P*(no sixes) = *P*(not a six and not a six and not a six)

= *P*(not a six) × *P*(not a six) × *P*(not a six)

=  = 0.579.

**Problem:** Interpret the probability that you calculated in the previous problem.

**Solution:** If you were to roll a die three times many, many times, about 57.9% of the time you would get no sixes in the three rolls.

**Problem:** If you roll a die three times, what is the probability that you get at least 1 six?

**Solution:** Because rolling at least 1 six is the complement of rolling no sixes,

*P*(at least 1 six) = 1 – *P*(no sixes) = 1 – 0.579 = 0.421.

**Problem:** If you roll a die three times, what is the probability that you get exactly 1 six?

**Solution:** There are three different ways this could occur:

* six and not six and not six

OR

* not six and six and not six

OR

* not six and not six and six

Because these outcomes are mutually exclusive, I can use the addition rule once I know the probability of each individual outcome. To find the probability of each outcome, I can use the multiplication rule because the rolls of the die are independent.

*P*(roll exactly 1 six) = 

***Page 567: The Last Chocolate***

Do people have a preference for the last thing they taste? Researchers at the University of Michigan designed a study to find out. The researchers gave 22 students five different Hershey’s Kisses (milk chocolate, dark chocolate, crème, caramel, and almond) in random order and asked the student to rate each one. Participants were not told how many Kisses they would taste. However, when the 5th and final Kiss was presented, participants were told that it would be their last one. Of the 22 students, 14 of them gave the final Kiss the highest rating.

<Source: <http://www.sitemaker.umich.edu/eob/files/obrienellsworth2012.pdf>>

**Problem:** If the participants in the study don’t have a special preference for the last Kiss they try, what is the probability that a participant picks the last chocolate?

**Solution:** Because the Kisses were presented in a random order, each participant’s favorite type is equally likely to be presented first, second, third, fourth, or last. Thus, the probability that a participant picks the last chocolate should be 1/5 = 0.20 (assuming that they don’t have a special preference for the last Kiss).

**Problem:** Explain why it is appropriate to use a binomial distribution to calculate the probability that exactly 14 of participants choose the last chocolate, assuming that they don’t have a preference for the last Kiss.

**Solution:** I can use the binomial distribution to answer this question because I am interested in *x* = the number of participants who choose the last chocolate (success) in a fixed number of attempts. Notice that the four binomial conditions are satisfied.

1. There are a fixed number of attempts (*n* = 22 participants).
2. There are two outcomes for each attempt (choose the last chocolate or don’t).
3. The choices of each participant are independent. That is, knowing which chocolate is preferred by one participant shouldn’t help predict the chocolate chosen by another participant.
4. The probability of success (*p* = 1/5 = 0.20) remains the same because there are always 5 chocolates to choose from.

**Problem:** Use the binomial probability formula to calculate the probability that exactly 14 of the 22 participants choose the last chocolate.

**Solution:** *P*(*x* = 14) = (22C14)(0.20)14(0.80)8 = 0.00000879.

**Problem:** Use technology to calculate the probability that *at least* 14 of the 22 participants choose the last chocolate.

**Solution:** *P*(*x* ≥ 14) = 1 – *P*(*x* ≤ 13) = 1 – binomcdf(*n* = 22, *p* = 0.20, *x* = 13)

= 1 – 0.99999 = 0.00001

**Problem:** Based on the probability you calculated in the previous problem, is there convincing evidence that people have a preference for the last thing they taste? Explain.

**Solution:** Yes. Because the probability of observing at least 14 people choose the last chocolate by random chance is so small, we have convincing evidence that people have a preference for the last thing they taste.

***Page 570: Does The Weather Channel Deliberately Make Mistakes?***

In his most recent book (*The Signal and the Noise: Why So Many Predictions Fail – But Some Don’t,* The Penguin Press 2012), Nate Silver claimed that the Weather Channel deliberately makes poor predictions in some instances. People tend to notice the failure to predict rain more than they do other mistakes. People don’t tend to mind an unexpected sunny day, but they very much mind having their picnic ruined by an unexpected rain storm.

When The Weather Channel states that there is a 20% chance of rain, it should actually rain 20% of the time in order for their predictions to be considered “calibrated.” However, a study by Texas A & M University found that out of 113 days that The Weather Channel predicted a 20% chance of rain, it only rained 6 times.

**Problem:** Calculate the expected number of rainy days out of the 113 days where The Weather Channel predicted a 20% chance of rain, assuming that their predictions are calibrated.

**Solution:** This is a binomial context where *n* = 113 and *p* = 0.20. The expected number of rainy days is *np* = 113(0.20) = 22.6 days.

**Problem:** Interpret the expected value from the previous problem.

**Solution:** If The Weather Channel were to make many sets of 113 predictions where there was a 20% chance of rain, the *average* number of rainy days would be about 22.6.

**SRIS Chapter 15 Alternate examples**

***Page 588: Finger Length***

Is there a relationship between gender and relative finger length? To find out, we used the random sampler at the United States Census At School Web site ([www.amstat.org/ censusatschool](http://www.amstat.org/%20censusatschool)) to randomly select 34 U.S. high school students who completed the Census at School survey. The data table shows the gender of each student and which finger was longer on their left hand (index finger, ring finger, or same length).

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Student** | **Gender** | **Longer Finger** | **Student** | **Gender** | **Longer Finger** |
| 1 | Female | Ring finger | 18 | Female | Ring finger |
| 2 | Female | Same length | 19 | Male | Ring finger |
| 3 | Male | Index finger | 20 | Female | Index finger |
| 4 | Male | Same length | 21 | Male | Ring finger |
| 5 | Female | Index finger | 22 | Male | Same length |
| 6 | Male | Ring finger | 23 | Male | Ring finger |
| 7 | Male | Ring finger | 24 | Male | Same length |
| 8 | Male | Ring finger | 25 | Female | Ring finger |
| 9 | Male | Same length | 26 | Female | Ring finger |
| 10 | Male | Ring finger | 27 | Male | Same length |
| 11 | Female | Index finger | 28 | Male | Ring finger |
| 12 | Male | Index finger | 29 | Male | Ring finger |
| 13 | Male | Ring finger | 30 | Female | Same length |
| 14 | Male | Index finger | 31 | Female | Index finger |
| 15 | Male | Ring finger | 32 | Male | Ring finger |
| 16 | Male | Index finger | 33 | Female | Ring finger |
| 17 | Female | Same length | 34 | Female | Ring finger |

**Problem:** Summarize the relationship between gender and relative finger length in a two-way table.

**Solution:** To create the two-way table, I will list the two genders across the top and the outcomes for relative finger length down the left side.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Male** | **Female** | **Total** |
| **Index finger** |  |  |  |
| **Ring finger** |  |  |  |
| **Same length** |  |  |  |
| **Total** |  |  |  |

Then, I will count the number of males who had a longer index finger. This happened 4 times (students 3, 12, 14, and 16). I will then count the number of students for each of the other 5 combinations and enter these values in the two-way table below.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Male** | **Female** | **Total** |
| **Index finger** | 4 | 4 | **8** |
| **Ring finger** | 12 | 6 | **18** |
| **Same length** | 5 | 3 | **8** |
| **Total** | **21** | **13** | **34** |

**Problem:** If you were to randomly select 1 of these 34 students, what is the probability that you select a person who is male and has a longer index finger?

**Solution:** Because there were 4 males with longer index fingers in the group of 34 students, *P*(male and index) = = 0.118.

**Problem:** Interpret the probability from the previous problem.

**Solution:** If we were to select a student at random from this group of students many, many times, we would get a person who is male and has a longer index finger about 11.8% of the time.

**Problem:** If you were to randomly select 1 of these 34 students, what is the probability that you select a male or a student with a longer index finger?

**Solution:** Using the general addition rule, *P*(male or index) = *P*(male) + *P*(index) – *P*(male and index) =  = 0.735.

**Problem:** If you were to randomly select 1 of these 34 students, what is the probability that you select neither a male nor someone who has a longer index finger?

**Solution:** Using the complement rule,

*P*(not male and not index) = 1 – *P*(male or index) = = 0.265.

***Page 592: Finger Length, part 2***

In the previous example, we began to investigate the relationship between gender and relative finger length for a random sample of 34 students. Here is the two-way table that summarizes these data:

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Male** | **Female** | **Total** |
| **Index finger** | 4 | 4 | **8** |
| **Ring finger** | 12 | 6 | **18** |
| **Same length** | 5 | 3 | **8** |
| **Total** | **21** | **13** | **34** |

**Problem:** If you randomly select a male, what is the probability that he has a longer index finger?

**Solution:** Because there are 21 males to choose from and 4 of them have a longer index finger, *P*(index | male) =  = 0.190.

**Problem:** If you randomly select a student with a longer index finger, what is the probability that he or she is a male?

**Solution:** Because there are 8 students with a longer index finger and 4 of them are males, *P*(male | index) =  = 0.500.

***Page 594: Finger Length, part 3***

In the previous two examples, we began to investigate the relationship between gender and relative finger length for a random sample of 34 students. Here is the two-way table that summarizes these data:

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Male** | **Female** | **Total** |
| **Index finger** | 4 | 4 | **8** |
| **Ring finger** | 12 | 6 | **18** |
| **Same length** | 5 | 3 | **8** |
| **Total** | **21** | **13** | **34** |

**Problem:** Explain what it means if the events “select a male” and “select a student with a longer index finger” are independent.

**Solution:** If these events are independent, then knowing whether or not we selected a male wouldn’t change the probability that we select a student with a longer index finger. In other words, the probability of selecting a student with a longer index finger is the same if we had selected a male or selected a female.

**Problem:** For these 34 students, are the events “select a male” and “select a student with a longer index finger” independent? Explain.

**Solution:** No, these events are not independent. The probability of selecting someone with a longer index finger is smaller when we select a male than when we select a female.

*P*(index | male) = 4/21 = 0.190 < *P*(index | female) = 4/13 = 0.308.

***Page 597: Late for School***

Shannon hits the snooze bar on her alarm clock on 60% of school days. If she doesn’t hit the snooze bar, there is a 0.90 probability that she makes it to class on time. However, if she hits the snooze bar, there is only a 0.70 probability that she makes it to class on time.

**Problem:** Express the provided information in probability notation.

**Solution:** *P*(snooze) = 0.60, *P*(on time | no snooze) = 0.90, *P*(on time | snooze) = 0.70.

**Problem:** Display the provided information using a tree diagram. Then, use the general multiplication rule to calculate the probabilities of the four possible outcomes.

**Solution:** The figure below shows the tree diagram for this chance process.

Probability

*P*(snooze and on time)

= 0.60 × 0.70 = 0.42

*P*(snooze and late)

= 0.60 × 0.30 = 0.18

0.70

0.30

0.90

0.10

0.40

0.60

Late

On time

Late

On time

No snooze

Snooze

*P*(no snooze and on time)

= 0.40 × 0.90 = 0.36

*P*(no snooze and late)

= 0.40 × 0.10 = 0.04

**Problem:** Calculate the probability that Shannon is late on a randomly selected day.

**Solution:** The probability that she hits the snooze bar and is late is (0.60)(0.30) = 0.18. The probability that she doesn’t hit the snooze bar and is late is (0.40)(0.10) = 0.04. Thus, *P*(late) = 0.18 + 0.04 = 0.22. Shannon is late on about 22% of school days.

**Problem:** Suppose that Shannon is late for school. What is the probability that she hit the snooze bar that morning?

**Solution:** We want to find

*P*(snooze | late) =  =  = 0.818.

When Shannon is late, there is a 0.818 probability that she hit the snooze bar that morning.

***Page 602: How Many Languages?***

Imagine selecting a U.S. high school student at random. Define the random variable *X* = number of languages spoken by the randomly selected student. The table below gives the probability distribution of *X*, based on a sample of students from the U.S. Census at School database.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Languages:** | 1 | 2 | 3 | 4 | 5 |
| **Probability:** | 0.630 | 0.295 | 0.065 | 0.008 | 0.002 |

**Problem:** Display the probability distribution of *X* with a graph. Briefly describe what you see.

**Solution:**  The histogram is shown below. Shape: The histogram is strongly skewed to the right. Center: The median is 1, but the mean will be slightly higher due to the skewness. Spread: The number of languages varies from 1 to 5, but nearly all of the students speak just one or two languages.



**Problem:** What is the probability that a randomly selected student speaks 3 or more languages?

**Solution:** *P*(*X*3) = *P*(*X* = 3) + *P*(*X* = 4) + *P*(*X* = 5) = 0.065 + 0.008 + 0.002 = 0.075. There is a 0.075 probability of randomly selecting a student who speaks 3 or more languages.

**Problem:** Compute the expected value of the random variable *X* and interpret this value in context.

**Solution:** = 1.457.

If we were to randomly select many, many U.S. high school students at random, the average number of languages spoken would be about 1.457.

***Page 605: High Deductible Fire Insurance***

Daren recently bought a cabin in the mountains and is considering two different options for fire insurance. The traditional, but more expensive, policy costs $600 per year and will completely cover the cost of rebuilding the cabin if it burns down in a forest fire. A cheaper policy only costs $100 per year, but includes a deductible of $100,000. That is, if the cabin is burned down Daren must pay $100,000 towards the cost and the insurance company will pick up the rest.

**Problem:** If Daren chooses the high deductible policy and his cabin burns down within the first year, what is his total cost?

**Solution:** Daren paid $100 for the policy and needs to pay $100,000 towards the cost of rebuilding the cabin. His total cost is $100 + $100,000 = $100,100.

**Problem:** Let *X* = Daren’s total cost if he buys the cheaper, high deductible insurance policy. Determine the probability distribution of *X*, using *p* for the probability that the cabin burns down.

**Solution:** Here is the probability distribution:

|  |  |  |
| --- | --- | --- |
| *x* | $100,100 | $100 |
| *P*(*x*) | *p* | 1 – *p* |

**Problem:** For what values of *p* would the cheaper, high deductible policy be a good idea?

**Solution:** To be a good strategy, the expected total cost of the high deductible policy must be *less* than the cost of the traditional policy. Thus,

100,100(*p*) + 100(1 – *p*) < 600

100,000*p* + 100 < 600

100,000*p* < 500



*p* < 0.005

As long as there is less than a 0.005 probability that the cabin burns down in the next year, the high deductible policy is a good idea.

***Page 610: Will You Go to Prom with Me?***

Steve wants to ask Ashleigh to prom, but he isn’t sure if she will say yes. To increase the chances that she will agree, he is considering a crazy idea: run out into the middle of the gym during an assembly, grab the microphone, and recite a poem that he wrote for Ashleigh. Of course, security might foil his plans and make him look like a weirdo. If he is successful at the assembly, Steve estimates that there is a 90% chance that Ashleigh will say yes. If he is unsuccessful, Steve estimates that there is only a 5% chance that she will say yes. If he asks her in a more traditional way (face-to-face after their statistics class), Steve estimates that there is a 30% chance that she will say yes.

**Problem:** Create a tree diagram for Steve’s assembly idea, using the possible outcomes of his attempt to recite the poem as the first set of branches and the possible outcomes of asking Ashleigh to prom as the second set of branches. Use *p* for the probability of successfully reciting the poem.

**Solution:** Here is the tree diagram:

Probability

Poem successful

Poem not successful

Ashleigh says yes

Ashleigh says no

Ashleigh says yes

Ashleigh says no

*p*

1 – *p*

0.95

0.05

0.10

0.90

*P*(poem successful and Ashleigh says yes)

= *p* × 0.90

*P*(poem successful and Ashleigh says no)

= *p* × 0.10

*P*(poem not successful and Ashleigh says yes)

= (1 – *p*)(0.05)

*P*(poem not successful and Ashleigh says no)

= (1 – *p*)(0.95)

Attempt

Poem

**Problem:** Using the tree diagram, calculate the probability that Ashleigh says yes, given that Steve attempts to read the poem at the assembly.

**Solution:** *P*(yes) = *P*(poem successful and yes) + *P*(poem not successful and yes)

= (*p* × 0.90) + (1 – *p*)(0.05)

= 0.85*p* + 0.05

**Problem:** For what values of *p* would it be worthwhile for Steve to attempt to read the poem at the assembly?

**Solution:** To be a worthwhile strategy, the probability that Ashleigh says yes after he attempts to read the poem at the assembly must be greater than the probability that she says yes if he asks her in the traditional way (0.30).

0.85*p* + 0.05 > 0.30

0.85*p* > 0.25

*p* > 0.294

If Steve has greater than a 29.4% chance of successfully reading the poem at the assembly, he should go with the risky strategy.